INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH AND ANALYSIS

ISSN(print): 2643-9840, ISSN(online): 2643-9875

Volume 09 Issue 11 November 2026

DOI: 10.47191/ijmra/v9-i11-31, Impact Factor: 8.685

Page No. 7801-7810

A Mathematical Model for Axle-Specific Thermal Load Prediction of Ultra-Large Off-the-Road Tires Considering Longitudinal Haul-Road Grade

R. Ergashev[1], G. Ubaydullayev[2], Q. Urinbayev[2], U. Yusupov[2*]

[1]Navoi Mining and Metallurgical Company JSC, Navoi, Uzbekistan

[2]Department of Vehicle Engineering, Tashkent State Transport University, Tashkent, Uzbekistan

Corresponding Author: U. Yusupov

ABSTRACT: Conventional ton-kilometer-per-hour (TKPH) procedures for ultra-large off-the-road (OTR) tires compute the mean tire load from a fixed static axle split and correct only for cycle length and ambient temperature, so the axle-load transfer caused by longitudinal haul-road grade is ignored. This study derives an axle-specific TKPH model from the rigid-body equilibrium of a haul truck on a grade, in which the loaded truck travels uphill and the empty truck downhill, and the mean tire load of each axle is computed from the grade-dependent normal reactions. Linearization of the model yields closed-form grade coefficients that depend only on vehicle mass, wheelbase and centre-of-gravity coordinates. The parameters were measured for four truck classes at the Muruntau open-pit mine (Uzbekistan). For a Komatsu 830E operating at 45 °C, a 10% grade changes the full-cycle TKPH by −11.3% on the front axle and +6.1% on the rear axle (−1.08% and +0.67% per grade point), and the TKPH of the loaded uphill leg by −28.2% and +13.2%. The linear correction factors reproduce the full model within 0.5% for grades up to 10%, and the predicted static axle split agrees with manufacturer data within 0.5 percentage points. After the method was introduced into the monitoring of 40.00R57 tires, the share of premature removals decreased from 62.1% to 45.6% ($p < 0.001$) and the mean removal mileage increased by 4.56%, although concurrent changes in the tire supplier mix prevent attributing these changes solely to the method. The model provides a transparent basis for axle-specific tire selection and route-sensitive thermal-risk assessment.

KEYWORDS: Off-the-road tire, TKPH, haul truck, longitudinal road grade, axle-load transfer, open-pit mine

I. INTRODUCTION

Ultra-large off-the-road (OTR) tires are among the most expensive and safety-critical components of mining haul trucks operating in large open-pit mines. Their service life is controlled not only by mechanical wear and impact damage but also by progressive thermal degradation caused by cyclic deformation of the tire structure. Excessive internal temperature accelerates rubber ageing, promotes belt and ply separation, and increases the probability of catastrophic tire failure. Consequently, prediction of tire thermal loading is an important engineering problem in mine haulage [1–6]. At the Muruntau open-pit mine of Navoi Mining and Metallurgical Company (NMMC), tires accounted for 27% of haul-truck operating costs in 2025, second only to fuel and lubricants (34%) [7].

Existing approaches to tire thermal assessment fall into three groups: finite-element thermomechanical simulations, field monitoring of cavity-air temperature or pressure (TPMS), and the ton-kilometer-per-hour (TKPH) index. Finite-element models give detailed temperature fields but require extensive material data and computing time [8–13]. The TKPH index, introduced in SAE J1015 and J1098, compares the product of mean tire load and mean operating speed with the rated capacity of the tire and remains the standard criterion for OTR tire selection [14, 15].

Manufacturers apply the TKPH concept in slightly different ways. Michelin corrects the site value with a cycle-length coefficient K_1 for cycles longer than 5 km and an ambient-temperature coefficient K_2 referred to 38 °C [16]; Bridgestone instead corrects the rated TKPH of the tire for ambient temperature and haul distance [15]; Yokohama publishes a rated TKPH (TMPH) for each tire to be compared with the site value [17]. In the Michelin procedure, as implemented in commercial dispatch software, the mean tire load of each axle is obtained from a fixed static split of the gross vehicle weight (33.3/66.7% for a loaded two-axle truck) [16], and

none of the published procedures contains a term for road grade. Recent refinements have focused on the same two coefficients: Ma et al. used a finite-element thermal model to recalibrate K_1 and K_2 for ultra-large tires [18, 19].

Road grade is the main operating factor missing from this framework. On a longitudinal grade the normal reactions are redistributed between the axles, so that the front and rear tires of the same truck carry different loads on the loaded and empty legs of the cycle. Field studies at quarries in Uzbekistan have shown that the average longitudinal route slope affects tire service life [14], and an expert assessment ranked it among the significant factors that the standard TKPH formula ignores [20]. Related work has shown that severe operating conditions shorten the life of haul-truck components [21] and that the reliability of mining equipment is a dominant research theme in the mining industry [22]. However, a closed-form, axle-resolved correction of the TKPH index for road grade has not been reported.

The present study fills this gap. Its contributions are: (i) an axle-specific TKPH model in which the mean tire load of each axle is computed from the grade-dependent normal reactions of the loaded truck travelling uphill and the empty truck travelling downhill; (ii) closed-form grade coefficients, obtained by linearizing the model, that depend only on vehicle mass, wheelbase and centre-of-gravity coordinates and can therefore be computed for any two-axle haul truck; (iii) verification of the model against manufacturer axle-load data and an error analysis of the linearized correction; and (iv) an evaluation of fleet-level tire-removal statistics at the Muruntau mine before and after the method was introduced into tire monitoring.

II. MATERIALS AND METHODS

A. Research Framework

The methodology is summarized in Fig. 1. The conventional TKPH procedure (Section II-B) is extended with a rigid-body axle-load model for longitudinal grades (Section II-C), which is used to compute an axle-specific TKPH (Section II-D). Linearization of this model gives closed-form grade coefficients (Section II-E). The model parameters and field data are described in Section II-F, and the verification and validation procedure in Section II-G.

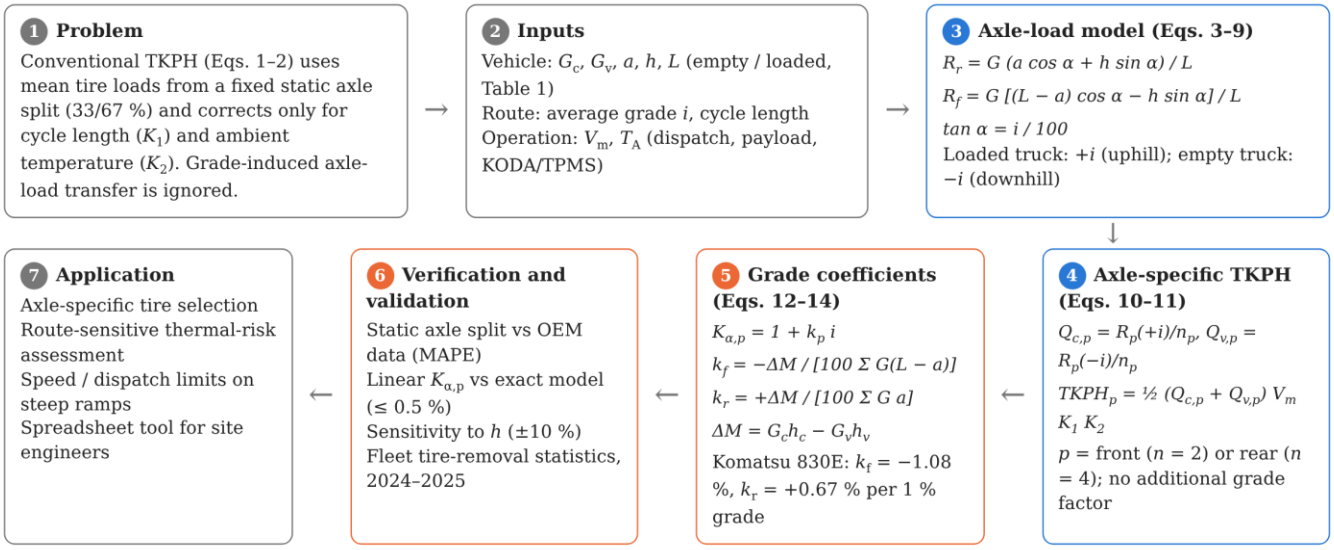


Fig. 1. Research framework of the proposed axle-specific TKPH model

B. Conventional TKPH Assessment

In the conventional procedure the actual (real site) TKPH of the tires on axle p is calculated as

$$\mathrm{TKPH}_p = Q_{m,p}\,V_m\,K_1\,K_2, \qquad Q_{m,p} = \frac{Q_{c,p} + Q_{v,p}}{2} \tag{1}$$

where $Q_{c,p}$ and $Q_{v,p}$ are the loads per tire on axle p of the loaded and empty truck (t), obtained from a fixed static axle split; V_m is the mean operating speed of the cycle (km/h); K_1 is the cycle-length coefficient (K_1 = 1 for cycles up to 5 km, increasing to 1.06 at 7 km and 1.12 at 10 km [16]); and K_2 is the ambient-temperature coefficient [16]:

$$K_2 = 1 + \beta\,\frac{T_A - T_B}{V_m}, \qquad \beta = \begin{cases} 0.25, & T_A < T_B \\ 0.40, & T_A \geq T_B \end{cases} \tag{2}$$

where T_A is the maximum shade temperature of the hottest period (°C) and T_B = 38 °C is the reference temperature. The tire is acceptable if TKPH_p does not exceed its rated TKPH. Equation 1 contains no term for road grade.

C. Axle Load Model for Mining Haul Trucks on Longitudinal Grades

The forces acting on a haul truck on a longitudinal grade are shown in Fig. 2. The truck is treated as a rigid body in quasi-static equilibrium; the gross vehicle weight G is expressed in tonnes, so that the axle reactions are obtained directly in tonnes. Taking moments about the front-axle contact point A gives

$$\sum M_A = 0: \quad R_r L - G\,h\sin\alpha - G\,a\cos\alpha = 0 \tag{3}$$

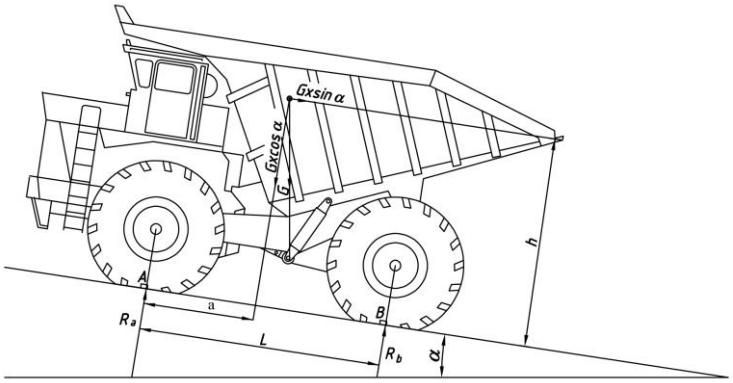


Fig. 2. Forces acting on a mining haul truck travelling on a longitudinal grade

from which the rear- and front-axle normal reactions are

$$R_r = \frac{G(a\cos\alpha + h\sin\alpha)}{L} \tag{4}$$

$$R_f = G\cos\alpha - R_r = \frac{G[(L-a)\cos\alpha - h\sin\alpha]}{L} \tag{5}$$

where a is the distance from the front axle to the centre of gravity (m), h is the height of the centre of gravity (m), L is the wheelbase (m) and α is the grade angle, positive when the truck travels uphill. Because the weight component normal to the road is $G\cos\alpha$, the reactions sum to $G\cos\alpha$ rather than G; writing $R_f = G - R_r$ would overestimate the front-axle load by $G(1-\cos\alpha)$, i.e. by 2.1% of the loaded front-axle load of a Komatsu 830E at a 10% grade. The grade angle is related to the grade i (%) by

$$\tan\alpha = \frac{i}{100}, \qquad \alpha = \arctan\left(\frac{i}{100}\right) \tag{6}$$

so that

$$\sin\alpha = \frac{i/100}{\sqrt{1+(i/100)^2}}, \qquad \cos\alpha = \frac{1}{\sqrt{1+(i/100)^2}} \tag{7}$$

Substituting Equation 7 into Equations 4 and 5 gives the reactions as explicit functions of grade:

$$R_r(i) = \frac{G(a + h\,i/100)}{L\sqrt{1+(i/100)^2}} \tag{8}$$

$$R_f(i) = \frac{G[(L-a) - h\,i/100]}{L\sqrt{1+(i/100)^2}} \tag{9}$$

Equations 8 and 9 are used with $i > 0$ for uphill and $i < 0$ for downhill travel, and with separate values of G, a and h for the loaded and empty truck. The values for the four investigated trucks are given in Table I (Section II-F).

Table I. Vehicle Geometry and Mass Parameters Used in The Axle-Load Model

Truck model	a, empty / loaded (m)	h, empty / loaded (m)	L (m)	Gross / empty mass (t)
BelAZ 7555	1.960 / 2.680	2.405 / 2.730	4.000	95.50 / 40.50
BelAZ 75131	2.597 / 3.551	3.760 / 4.420	5.300	245.50 / 109.50
CAT 793D	3.188 / 3.956	4.710 / 5.520	5.905	383.75 / 165.75
Komatsu 830E	3.175 / 4.254	4.940 / 5.840	6.350	385.85 / 164.20

D. Axle-specific TKPH Model

In open-pit haulage the truck normally climbs loaded from the pit and descends empty. For a route with average grade i, the loads per tire are therefore taken from Equations 8 and 9 evaluated at $+i$ for the loaded truck (superscript c) and at $-i$ for the empty truck (superscript v):

$$Q_{c,p} = \frac{R_p^c(+i)}{n_p}, \qquad Q_{v,p} = \frac{R_p^v(-i)}{n_p} \tag{10}$$

where $n_f = 2$ and $n_r = 4$ are the numbers of tires on the front and rear axles. The axle-specific TKPH is then

$$\mathrm{TKPH}_p(i) = \frac{Q_{c,p} + Q_{v,p}}{2} V_m K_1 K_2, \qquad p \in \{f, r\} \tag{11}$$

At $i = 0$, Equation 11 reduces exactly to the conventional Equation 1. Because the grade enters Equation 11 through the axle loads, no additional grade factor is applied; multiplying Equation 11 by a grade coefficient would count the grade effect twice.

E. Closed-form Grade Coefficients

For practical use it is convenient to express the grade effect as a multiplier on the level-road TKPH. For $|i| \le 10\%$, $\cos\alpha \ge 0.995$ and $\sin\alpha$ differs from $i/100$ by less than 0.5%, so Equations 8–11 can be linearized about $i = 0$. Differentiating the mean tire load of each axle with respect to i and dividing by its level-road value gives the relative change of TKPH per 1% of grade:

$$k_f = -\frac{G_c h_c - G_v h_v}{100[G_c(L - a_c) + G_v(L - a_v)]} \tag{12}$$

$$k_r = \frac{G_c h_c - G_v h_v}{100(G_c a_c + G_v a_v)} \tag{13}$$

and the axle-specific grade correction factor

$$K_{\alpha,p} = 1 + k_p\, i, \qquad \mathrm{TKPH}_p(i) \approx \mathrm{TKPH}_p(0)\, K_{\alpha,p} \tag{14}$$

The numerator $G_c h_c - G_v h_v$ is the difference between the overturning moments of the loaded and empty truck; it is positive for all trucks because the payload raises both the weight and the centre of gravity. Hence $k_f < 0$ and $k_r > 0$: grade relieves the front tires and loads the rear tires over a complete cycle. The coefficients depend only on vehicle parameters, which removes the need for an empirical regression. Table II lists them for the four trucks of Table I, together with the corresponding single-leg coefficients ($-h/(L-a)$ and $+h/a$ per 100% of grade for the front and rear axles of a truck travelling uphill).

Table II. Grade Coefficients of The Axle-Specific TKPH (% Change Per 1% of Grade), Equations 12–13

Truck model	Cycle, front k_f	Cycle, rear k_r	Loaded uphill, front	Loaded uphill, rear	Empty downhill, front	Empty downhill, rear
BelAZ 7555	−0.78	+0.49	−2.07	+1.02	+1.18	−1.23
BelAZ 75131	−0.93	+0.58	−2.53	+1.24	+1.39	−1.45
CAT 793D	−1.12	+0.65	−2.83	+1.40	+1.73	−1.48
Komatsu 830E	−1.08	+0.67	−2.79	+1.37	+1.56	−1.56

Single-leg coefficients refer to a 1% change of grade on the loaded uphill leg or the empty downhill leg alone; cycle coefficients combine both legs according to Equation 11.

F. Study Site, Investigated Vehicles and Data

The study was carried out at the Muruntau open-pit mine (NMMC, Uzbekistan), where Komatsu, Caterpillar and BelAZ trucks operate on routes with sustained grades and summer shade temperatures above 40 °C. The axle loads of four truck models were measured on axle scales in level and raised positions, and the centre-of-gravity heights were calculated from the change of the front-axle load. The resulting parameters are given in Table I.

Operating data (cycle length, cycle time, payload and ambient temperature) were obtained from the mine dispatch system, the onboard payload monitoring systems and the KODA and TPMS systems installed on the investigated trucks. Three field campaigns were used: a BelAZ-7513 truck on three routes, two Komatsu 860E trucks with different tread compounds, and a Komatsu 830E truck (fleet No. 701) monitored for one month. Fleet-level tire-removal records for 40.00R57 tires in 2024 and the first seven months of 2025 were taken from the mine tire-management database.

G. Verification and Validation Procedure

The model was implemented in a spreadsheet tool and, independently, in a Python script; the two implementations agree to within 1 t·km/h. Its accuracy was assessed with three tests. (1) The static axle split predicted by Table I ($i = 0$) was compared with

manufacturer specification data for the Komatsu 830E-AC and Caterpillar 793D [23, 24], which are independent of the authors' measurements. (2) The linearized factor of Equation 14 was compared with the exact model of Equations 8–11 over $0 \le i \le 10\%$. (3) The sensitivity of the predicted TKPH to a ±10% error in the centre-of-gravity height was evaluated. Errors were quantified by the mean absolute percentage error and the root-mean-square error:

$$\mathrm{MAPE} = \frac{100}{N}\sum_{j=1}^{N}\left|\frac{y_j - \hat{y}_j}{y_j}\right| \tag{15}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(y_j - \hat{y}_j\right)^2} \tag{16}$$

where y_j is the reference value, $\hat{y}_j$ the model value and N the number of comparisons.

III. RESULTS AND DISCUSSION

A. Operational Assessment of Actual TKPH Performance

On 15 September 2025, 15 operating cycles of a BelAZ-7513 truck were recorded on three routes: A→B→A (3 cycles, one-way distance 6.5 km), C→D→C (6 cycles, 3.0 km) and E→F→E (6 cycles, 5.1 km). The truck was fitted with 33.00R51 ET304 R1 tires with a rated TKPH of 560 (Fig. 3).

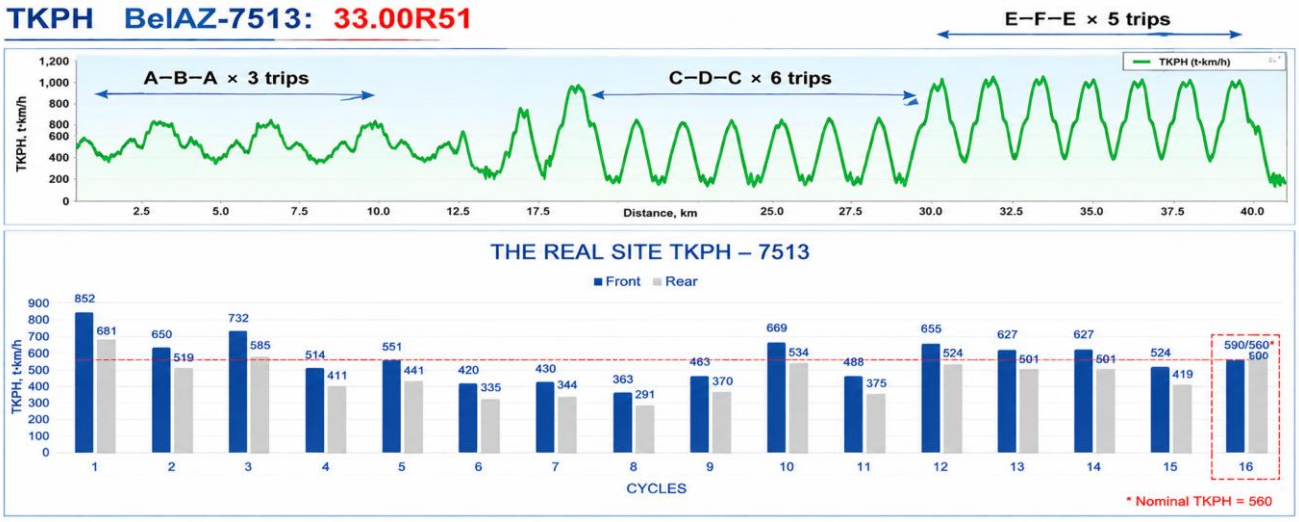


Fig. 3. Actual TKPH of the BelAZ-7513 mining haul truck tires on three routes

The tires operated within the permissible range on all routes, but values above the rated TKPH occurred on the longer routes and mainly on the front axle. To keep the actual TKPH close to the rating, a set of operational measures was developed, and the use of Techking tires on the rear axle during the summer season was recommended.

B. Effect of Tire Specification on Thermal Loading

During 14–17 June 2025, two Komatsu 860E trucks (No. 383 and No. 385) fitted with Bridgestone 50/80R57 VREV E2A (cut-resistant) and VREV E1A (standard) tires, respectively, were monitored under comparable conditions, each over 57 haul cycles; the maximum shade temperature reached 44 °C. With an average payload of 218 t and a one-way distance of 5.2 km, nearly 50% of the cycles of truck No. 383 exceeded the rated TKPH of the E2A tire, whereas only 19% of the cycles of truck No. 385 (215 t, 4.8 km) exceeded the rating of the E1A tire (Fig. 4). The E1A specification was therefore recommended for the investigated conditions.

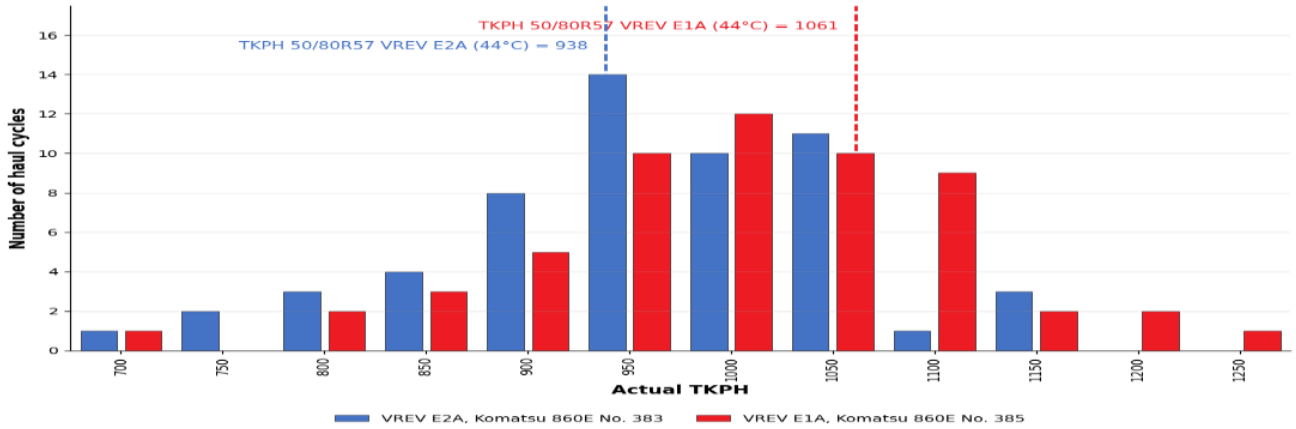


Fig. 4. Distribution of actual TKPH for Komatsu 860E trucks No. 383 (E2A tires) and No. 385 (E1A tires) at the Muruntau mine

Both campaigns show that the governing axle and the margin to the rated TKPH depend on the route; this motivates the axle-specific treatment of grade developed in Section 2.

C. Effect of Longitudinal Grade on Axle-specific TKPH

The Komatsu 830E truck (fleet No. 701) was monitored in June 2025 (Fig. 5). The operating parameters used in the model are the mean cycle speed V_m = 15.4 km/h, a cycle length of 7.43 km (K_1 = 1.06) and T_A = 45 °C, which gives K_2 = 1.182 by Equation 2. The truck geometry and masses are those of Table I. Table III and Fig. 6 show the axle-specific TKPH predicted by Equations 8–11 for average route grades of 0–10%.

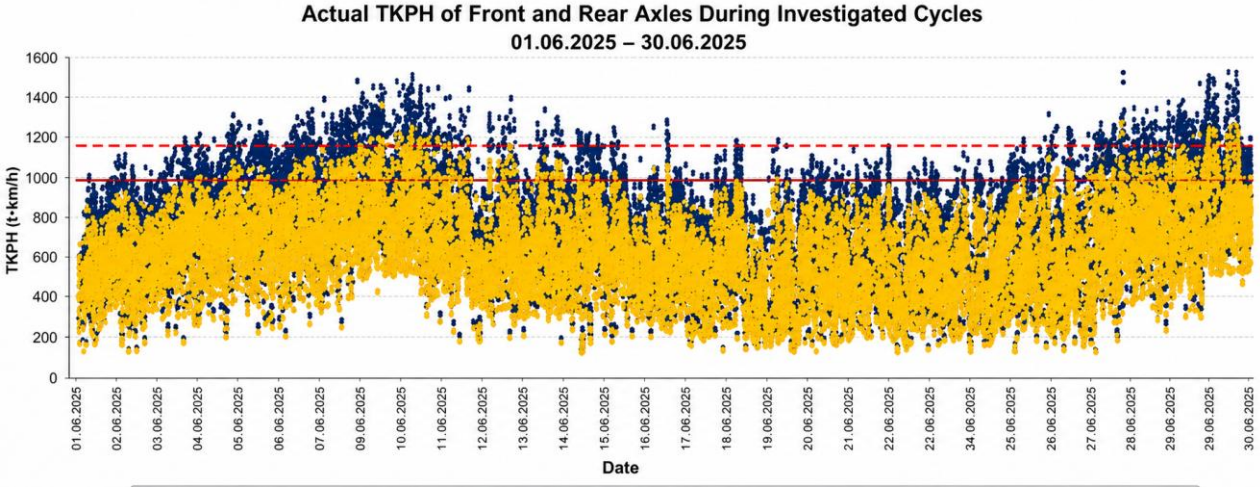


Fig. 5. Actual TKPH values of the front and rear axles of the Komatsu 830E truck during the monitored cycles

Table III. Axle-specific TKPH (t·km/h) of the Komatsu 830E predicted by Equations 8–11 and by the linear factor of Equation 14

Grade *i* (%)	Loaded uphill, front	Loaded uphill, rear	Empty downhill, front	Empty downhill, rear	Cycle, front (Eq. 11)	Cycle, rear (Eq. 11)	Cycle, front (Eq. 14)	Cycle, rear (Eq. 14)
0	1229	1247	792	396	1010	821	1010	821
2	1160	1281	816	384	988	832	988	832
4	1091	1314	841	371	966	843	966	843
6	1021	1347	864	358	943	853	945	854
8	952	1379	888	346	920	862	923	865
10	882	1411	911	333	896	872	901	876
Change at 10%	**−28.2%**	**+13.2%**	**+15.0%**	**−16.0%**	**−11.3%**	**+6.1%**	**−10.8%**	**+6.7%**

V_m = 15.4 km/h, K_1 = 1.06, K_2 = 1.182. At *i* = 0 all values coincide with the conventional method (Eq. 1). Leg values are per-leg TKPH computed with the load of that leg only.

On the loaded uphill leg the front-axle TKPH decreases from 1229 to 882 t·km/h (−28.2%) as the grade increases to 10%, while the rear-axle TKPH increases from 1247 to 1411 t·km/h (+13.2%). On the empty downhill leg the trend is reversed: the front axle gains +15.0% and the rear axle loses 16.0%. The rear tires of the loaded truck are therefore the most heavily loaded tires on steep ramps, which agrees with the higher incidence of thermal damage and ply separation of rear tires on uphill haul routes.

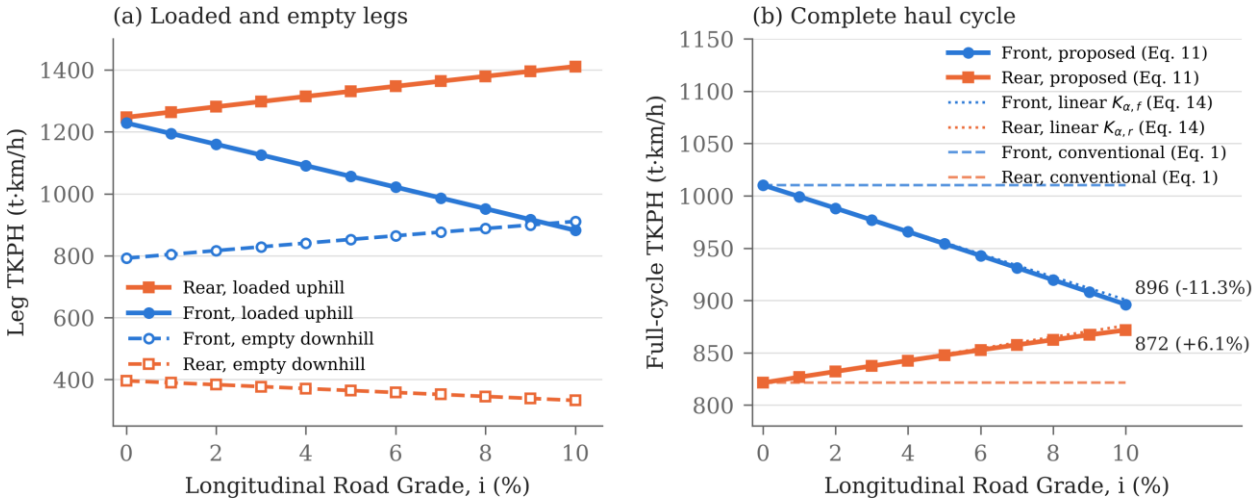


Fig. 6. Effect of longitudinal road grade on the axle-specific TKPH of the Komatsu 830E: (a) loaded uphill and empty downhill legs; (b) complete haul cycle, proposed model (Eq. 11), linear factor (Eq. 14) and conventional method (Eq. 1)

Over a complete cycle the two legs partly compensate each other. At a 10% grade the front-axle TKPH is 11.3% lower and the rear-axle TKPH 6.1% higher than the conventional values (1010 and 821 t·km/h). These changes correspond to the closed-form coefficients k_f = −1.08% and k_r = +0.67% per 1% of grade (Table II), so that for the Komatsu 830E Equation 14 becomes $K_{\alpha,f}$ = 1 − 0.0108 i and $K_{\alpha,r}$ = 1 + 0.0067 i. For the four trucks studied, k_f ranges from −1.12% to −0.78% and k_r from +0.49% to +0.67% per 1% of grade; the coefficients are therefore truck-specific and should be computed from Equations 12 and 13 rather than transferred between models.

In practice the conventional method thus overestimates the front-axle TKPH and underestimates the rear-axle TKPH on graded routes. Since the conventional procedure rates the tire by the more heavily loaded axle, it can lead to the selection of an unnecessarily heat-resistant front tire while the thermal margin of the rear tires on ramps is overestimated.

D. Model Verification and Validation

The static axle split predicted from Table I is compared with manufacturer data in Table IV. The largest deviation, 0.5 percentage points, occurs for the empty Komatsu 830E (50.0/50.0% predicted against 49.5/50.5% specified); the mean absolute deviation over the four cases is 0.13 percentage points. For the Komatsu 830E-AC the predicted loaded axle loads (127.4 t front, 258.5 t rear) differ from the specified values (127.3 t and 258.5 t) by less than 0.03%.

Table IV. Static Axle-Load Split (Front / Rear, %) Predicted from Table I And Specified by the Manufacturers

Truck, condition	Manufacturer data	Model (i = 0)	Abs. deviation (pp)
Komatsu 830E, empty	49.5 / 50.5	50.0 / 50.0	0.50
Komatsu 830E, loaded	33.0 / 67.0	33.0 / 67.0	0.01
CAT 793D, empty	46.0 / 54.0	46.0 / 54.0	0.01
CAT 793D, loaded	33.0 / 67.0	33.0 / 67.0	0.01

Manufacturer data: Komatsu 830E-AC and Caterpillar 793D specification sheets [23, 24].

The linear factor of Equation 14 reproduces the exact model of Equation 11 with MAPE = 0.17% and a maximum error of 0.50% for both axles over 0–10% grade (Table III). A ±10% error in the centre-of-gravity height changes the full-cycle TKPH at a 10% grade by ∓1.2% on the front axle and ±0.6% on the rear axle, which is small compared with the grade effect itself (−11.3% and +6.1%). Using $R_f = G - R_r$ instead of Equation 5 would overestimate the front-axle cycle TKPH at 10% grade by 1.5% (loaded leg: 2.1%).

These tests verify the static parameters and the numerical implementation; the grade-dependent part of the model follows from rigid-body statics and its accuracy is governed mainly by the centre-of-gravity height, whose influence was bounded above.

A direct validation of the grade term against axle-resolved tire temperatures requires synchronized KODA temperature records and route-grade data for individual cycles and is planned as the next stage of the work.

E. Fleet-level Results of Implementation

The axle-specific method was introduced into the TKPH monitoring of 40.00R57 tires at the Muruntau mine in 2025. Table V compares tire-removal records for 2024 and the first seven months of 2025. Premature removals are those caused by mechanical damage or separation, as opposed to removal at the wear limit.

Table V. Removals of 40.00R57 tires at the Muruntau mine by cause, 2024 and January–July 2025

Indicator	2024	Jan–Jul 2025	Change
Tires removed	1333	1286	–
Mechanical damage, n (%)	497 (37.3)	420 (32.7)	−4.6 pp
Separation, n (%)	331 (24.8)	166 (12.9)	−11.9 pp
Worn out, n (%)	505 (37.9)	700 (54.4)	+16.5 pp
Premature removals, n (%)	**828 (62.1)**	**586 (45.6)**	**−16.5 pp**
Mean removal mileage (km)	51,361	53,705	+4.56%

Premature removals = mechanical damage + separation. pp = percentage points.

The share of premature removals fell from 62.1% to 45.6%, a reduction of 16.5 percentage points (relative reduction 26.6%; 95% confidence interval 12.8–20.3 percentage points). A two-proportion z-test gives $z = 8.49$ ($p < 0.001$). The largest change was in separation failures, whose share almost halved (24.8% to 12.9%); separation is the failure mode most directly associated with excessive internal temperature and is therefore the one expected to respond to TKPH-based control. The mean mileage of removed tires increased from 51,361 to 53,705 km (+4.56%).

These results are observational. The supplier mix of 40.00R57 tires and the fleet size changed between the two periods, and the 2025 data cover only seven months, so the improvement cannot be attributed solely to the proposed method. A controlled comparison of trucks monitored with and without the axle-specific method on the same routes would be required to isolate its effect.

F. Comparison with Manufacturer Methods and Previous Studies

Table VI compares the proposed model with the TKPH procedures of tire manufacturers and with recent studies. All existing procedures correct the TKPH for cycle length and ambient temperature, and those that distinguish axles do so with a fixed static load split. The proposed model is the only one in which the axle loads, and therefore the governing axle, depend on the route grade, and it provides the grade correction in closed form from vehicle parameters.

Table VI. Comparison of The Proposed Model with Existing TKPH Procedures

Method	Axle loads	Cycle length	Ambient temperature	Road grade	Basis of correction
Michelin [16]	Per axle, fixed static split	K_1 table (> 5 km)	K_2, Eq. 2	Not included	Empirical tables
Bridgestone [15]	Mean tire load	Rating +12% below 5 km; ≤ 16 km	Correction of rated TKPH	Not included	Empirical coefficients
Yokohama [17]	Not stated in catalogue	Not stated in catalogue	Rating at 38 °C	Not included	Rated TKPH (TMPH)
Ma et al. [18, 19]	Mean tire load	Refined K_1	Refined K_2	Not included	Finite-element thermal model
Mukhitdinov et al. [14]	–	–	–	Average route slope vs tire life	Field statistics
This study	**Per axle, grade-dependent (Eqs. 8–10)**	***K*₁**	***K*₂**	**Explicit, k_p from Eqs. 12–13**	**Rigid-body statics, closed form**

G. Limitations

The model is quasi-static and neglects inertial load transfer during acceleration and braking, dynamic loads due to road roughness and lateral load transfer on curves. It assumes that the truck climbs loaded and descends empty; routes with loaded downhill haulage require the opposite sign of i in Equation 10. The average route grade is used, so short steep ramps within a cycle are smoothed. Finally, the thermal response of the tire is represented by TKPH rather than by measured temperature, and the direct temperature validation noted in Section III-D remains to be completed.

IV. CONCLUSIONS

1. An axle-specific TKPH model was derived from the rigid-body equilibrium of a haul truck on a longitudinal grade. The mean tire load of each axle is computed from the grade-dependent normal reactions of the loaded truck travelling uphill and the empty truck travelling downhill; at zero grade the model reduces to the conventional TKPH formula. The front-axle reaction was corrected to account for the normal weight component $G \cos \alpha$, and no additional grade factor is applied to the axle-specific TKPH.

2. Linearization of the model gives closed-form grade coefficients that depend only on vehicle mass, wheelbase and centre-of-gravity coordinates. For the Komatsu 830E they are k_f = −1.08% and k_r = +0.67% per 1% of grade; for the four trucks studied k_f ranges from −1.12% to −0.78% and k_r from +0.49% to +0.67%.

3. For a Komatsu 830E at 45 °C, a 10% grade changes the TKPH of the loaded uphill leg by −28.2% (front) and +13.2% (rear), and the full-cycle TKPH by −11.3% and +6.1%. The conventional method therefore overestimates the thermal loading of the front tires and underestimates that of the rear tires on graded routes.

4. The static axle split predicted by the model agrees with manufacturer data within 0.5 percentage points, the linear correction factor reproduces the exact model within 0.5% for grades up to 10%, and a ±10% error in the centre-of-gravity height changes the predicted cycle TKPH by at most 1.2%.

5. After the method was introduced into the tire monitoring of 40.00R57 tires at the Muruntau mine, the share of premature removals decreased from 62.1% to 45.6% ($p < 0.001$), mainly through fewer separation failures, and the mean removal mileage increased by 4.56%. Because the tire supplier mix and fleet size also changed, a controlled comparison and direct validation against axle-resolved tire temperatures are recommended as future work.

ACKNOWLEDGMENT

This research was conducted at the Department of Vehicle Engineering, Tashkent State Transport University. The authors thank the management and technical staff of the Muruntau open-pit mine of Navoi Mining and Metallurgical Company for supporting the field investigations and providing access to the operational and monitoring data. The research received no external funding. R. Ergashev is employed by Navoi Mining and Metallurgical Company, where the field data were collected; the authors declare no other conflicts of interest. The data and the calculation spreadsheet are available from the corresponding author upon reasonable request. Claude (Anthropic) was used for language editing and formatting; the authors reviewed the content and take full responsibility for it.

REFERENCES

1) Ma, S., Huang, G., Obaia, K., Moon, S. W., and Liu, W. V. 2022. Hysteresis loss of ultra-large off-the-road tire rubber compounds based on operating conditions at mine sites. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 236(2–3), 439–450. https://doi.org/10.1177/09544070211015525
2) Mujdeci, A., Marchante Rodriguez, V., Maqbool, H., Amadeo, A., and Grasso, M. 2025. Advances in numerical modelling of tyre fatigue performance: A review. Journal of Materials Science: Materials in Engineering, 20, 105. https://doi.org/10.1186/s40712-025-00315-7
3) Shakenov, A., Sładkowski, A., and Stolpovskikh, I. 2022. Haul road condition impact on tire life of mining dump truck. Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 6, 25–29. https://doi.org/10.33271/nvngu/2022-6/025
4) Fathi, H., El-Sayegh, Z., Ren, J., and El-Gindy, M. 2024. Analysis of tire–road interaction: A literature review. Machines, 12(11), 812. https://doi.org/10.3390/machines12110812
5) Yusupov, U., and Shavkatov, X. 2024. Development of a standard mileage for large-sized tires under moderately severe quarry conditions. BIO Web of Conferences, 141, 04034. https://doi.org/10.1051/bioconf/202414104034
6) Kuvondik, U., Zilola, R., and Mashrab, I. 2025. Improvement of the classification of causes for the decommissioning of large-size tires. Vibroengineering Procedia. https://doi.org/10.21595/vp.2025.24964

7) Boykov, V., and Israilov, M. 2025. Optimizing TKPH for large-sized tires in quarry conditions. Lecture Notes in Networks and Systems. Springer, Cham. https://doi.org/10.1007/978-3-031-95649-2_33
8) Marais, J., and Venter, G. 2018. Numerical modelling of the temperature distribution in the cross-section of an earthmover tyre. Applied Mathematical Modelling, 57, 360–375. https://doi.org/10.1016/j.apm.2018.01.018
9) Nyaaba, W., Frimpong, S., and Anani, A. 2019. Fatigue damage investigation of ultra-large tire components. International Journal of Fatigue, 119, 247–260. https://doi.org/10.1016/j.ijfatigue.2018.07.009
10) Hyttinen, J., Österlöf, R., Jerrelind, J., and Drugge, L. 2024. A semi-physical thermodynamic transient rolling resistance model with nonlinear viscoelasticity. Mechanics of Time-Dependent Materials, 28(3), 895–916. https://doi.org/10.1007/s11043-023-09650-8
11) Hyttinen, J., Österlöf, R., Drugge, L., and Jerrelind, J. 2023. Constitutive rubber model suitable for rolling resistance simulations of truck tyres. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 237(1), 174–192. https://doi.org/10.1177/09544070221074108
12) Nguyen, T. C., Le, T. M. H., and Nguyen, H. H. 2023. Rolling tires on the flat road: Thermo-investigation with changing conditions through numerical simulation. Applied Sciences, 13(8), 4834. https://doi.org/10.3390/app13084834
13) Aldhufairi, H. S., and Olatunbosun, O. A. 2018. Developments in tyre design for lower rolling resistance: A state of the art review. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 232(14), 1865–1882. https://doi.org/10.1177/0954407017727195
14) Mukhitdinov, A., Tukhtamishov, S., and Urinbayev, Q. 2024. Results of the study of the influence of an average longitudinal slope of routes on the life of tires in the quarry. AIP Conference Proceedings. https://doi.org/10.1063/5.0197301
15) Bridgestone Corporation. 2024. Off-the-Road Tire Data Book. Bridgestone Corporation, Tokyo, Japan.
16) Micromine. 2024. How is the site TKPH calculated in Haul Infinity? (implementation of the Michelin real site TKPH procedure). Micromine Alastri Help Centre. https://micromine-alastri.zendesk.com/hc/en-us/articles/28334148954393
17) Yokohama Off-Highway Tires. 2021. Yokohama OTR Product Catalog (YPL297). Yokohama Off-Highway Tires America.
18) Ma, S., Fan, C., and Liu, W. V. 2024. Effects of site operating conditions on real site TKPH (tonne-kilometer-per-hour) of ultra-large off-the-road tires. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering. https://doi.org/10.1177/09544070231166166
19) Ma, S. 2023. Refining real site TKPH (tonne-kilometer-per-hour) of ultra-large off-the-road tires at mine sites. Doctoral Thesis. University of Alberta, Edmonton.
20) Israilov, M., and Yusupov, U. 2025. Determining the weight of factors affecting the TKPH of large-sized tires. E3S Web of Conferences, 627, 05010. https://doi.org/10.1051/e3sconf/202562705010
21) Botyan, E., Lavrenko, S., and Pushkarev, A. 2024. Evaluation of complicated mining exploitation conditions influence on service life of open pit trucks suspensions with remote monitoring systems. International Journal of Engineering, Transactions B: Applications, 37(11), 2268–2275. https://doi.org/10.5829/ije.2024.37.11b.12
22) Babyr, N. V. 2024. Topical themes and new trends in mining industry: Scientometric analysis and research visualization. International Journal of Engineering, Transactions B: Applications, 37(2), 439–451. https://doi.org/10.5829/ije.2024.37.02b.18
23) Komatsu. 2013. 830E-AC Electric Drive Truck: Specifications. Komatsu America Corp.
24) Caterpillar. 2012. 793D Mining Truck: Specifications. Caterpillar Inc., Peoria, IL.

There is an Open Access article, distributed under the term of the Creative Commons Attribution – Non-Commercial 4.0 International (CC BY-NC 4.0) (https://creativecommons.org/licenses/by-nc/4.0/), which permits remixing, adapting and building upon the work for non-commercial use, provided the original work is properly cited.