

Application of Graphical Evaluation and Review Technique to Determine Product Completion Time in Manufacturing Process

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ABSTRACT: The discrepancy between actual production process time and standard time is often occurs, because the calculation of production time does not take into account the repair time caused by defective products. The Graphical Evaluation and Review Technique (GERT) method is used to determine the production process time by considering repair time due to defective product. GERT is used to calculate the average and variance of the production time in stochastic work network and put into consideration the possibility of rework so that the completion time is close to the actual condition. The calculation results show a difference between the standard production time of the front spar component of 3,841.69 minutes/unit and the rear spar of 4,080.81 minutes/unit, while based on the GERT method the production time of the front spar is 4,657.77 minutes/unit and the rear spar is 4,406.01 minutes/unit.

KEYWORDS: production process time, rework, standard time, stochastic work network, Graphical Evaluation and Review Technique,

I. INTRODUCTION

The problem of delays in product manufacturing process within the manufacturing industry is the crucial aspect that requires attention. Timely production time will have a positive and significant impact on customer satisfaction and loyalty (Yeronica and Laulita, 2024). This implies that products that delivered late to customers can lead to increase operational cost, disruptions in the supply chain, and ultimately affect the company performance and competitiveness. The manufacturing process is well-developed, starting with the fabrication and assembly, which strictly follows detailed procedures, along with the precise parameter calculation for both manual and automated processes. Every process is documented to meet the administrative requirements related to quality aspects. All of these efforts are done to make sure the component function, structural integrity, and product operating systems deliver reliable performance, thereby fulfilling customer requirement (Sarh et al., 2009).

The success of a production is determined by the accuracy of completing each stage of the work process. Delays in production caused by errors in determining the production completion time will impact the distribution schedules and reduce customer trust. One of the causes of production time discrepancies is the failure of companies to account for defect probabilities and rework time. The planned completion time of a product will deviate, resulting in errors in production scheduling. Companies need to review the product completion times by taking into account the probability of defects and rework time so that they can determine the appropriate time for production of a product. The stochastic network scheduling technique is commonly accepted as a potent tool for computing the completion time of intricate specialized projects (Nwadigo et al., 2021). The application of the GERT method can be used to determine solution to the problems related to production process completion time by calculating the probability of defects and rework time (Fitria et al., 2018). Integrating Graphical Evaluation and Review Technique (GERT) and Fuzzy Logic (FL) to create Fuzzy-based GERT (F-GERT) for scheduling construction projects (Pregina and Kannan, 2024). F-GERT utilizes fuzzy operations to handle uncertainty in activity durations. GERT networks are increasingly powerful tool for modeling, scheduling, planning, controlling, and analyzing of agricultural mechanization projects (Abdi et al., 2010).

II. METHODOLOGY

Graphical Evaluation and Review Technique (GERT) is a procedure or method that combine several theoretical disciplines such as flowgraph, Moment Generating Functions (FGM), and PERT (Program Evaluation and Review Technique). The implementation of the GERT method in the manufacturing sector can be used to solve problems related to production time which

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is not fixed for each process so that the completion time is always changing (Whitehouse, 1973). The GERT method has several implementation stages, as follows:

A. *Determining the Distribution shape of Each Activity.*

Using data plots (time) for every activity, so that certain distribution shape can be identified (Jose, 2012) (De Marco, 2018). The testing method that is used is a nonparametric statistical test, namely the Kolmogorov-Smirnov test. The resulting distribution shape are normal, constant, and exponential distributions.

B. *Determining GERT Parameters*

The GERT parameters are determined after identifying the distribution shape of each activity. The GERT parameters are used to obtain the Fungsi Generator Momen (FGM) equation.

C. *Menentukan Fungsi Generator Momen (FGM).*

FGM that is used for every activity is made in the form of a continuous random variable equation and a discrete random variable (Walpole dkk., 2012).

If x is a continuous random variable, then:

$$M(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} \cdot f(x) dx$$

If x is a discrete random variable, then:

$$M(t) = E(e^{tx}) = \sum_{i=1}^n e^{tx} \cdot P(x_i)$$

where:

$M(t)$ = Fungsi Generator Momen

$E(e^{tx})$ = Expected value of e^{tx}

x = Random variable

The probability functions for continuous and discrete random variables are:

1. Normal distribution

$$M(t) = E(e^{tx}) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

First moment

$$E(x) = \bar{x}$$

Then the moment generator function of the normal distribution is:

$$M(t) = E(e^{tx}) = e^{\bar{x}t + \frac{1}{2}\sigma^2 t^2}$$

2. Exponential distribution

$$M(t) = E(e^{tx}) = (1 - \frac{1}{a}t)^{-1}$$

First moment:

$$E(x) = \bar{x} = \frac{1}{a}$$

Then the moment generator function of the exponential distribution:

$$M(t) = (1 - \bar{x}t)^{-1}$$

3. Constant distribution

$$M(t) = E(e^{tx}) = e^{xt}$$

First moment:

$$E(x) = \bar{x} = x$$

Then the moment generator function of the constant distribution:

$$M(t) = e^{\bar{x}t}$$

D. *Determining W Function*

The W function is a transmittance parameter in the GERT network which is obtained from the result of multiplying the activity probability and the fungsi generator momen of that activity.

$$W_{ij}(t) = P_{ij} \cdot M_{ij}(t)$$

Where:

$W_{ij}(t)$ = W function as the transmittance parameter from node i to node j

P_{ij} = Probability of realization from node i to node j

$M_{ij}(t)$ = Fungsi generator momen activity from node i to node j

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E. Determining Loop, Topological Equation, Mean (μ) dan Variance (σ^2)

A path is a series of branches that connect two nodes and does not pass through the same node again. A loop is a series of branches that starts from a node and eventually returns to that same node without passing through any other node. A first order loop is a sequential path that starts from a node and will return to that node. An n-th order loop is a set of n first order loops that do not touch each other (Whitehouse, 1973). The topological equation method that can be used is:

1. The equation for a closed signal flow graph is:

$$H = 1 - L_1 + L_2 - L_3 + \dots + (-1)^i L_i + \dots = 0$$

where : L_i = the sum of all loops of order i.

2. Open Signal Flow Graph:

The equation for a closed network, which is also known as Mason's rule, is as follows:

$$H = 1 - \sum T(L_1) + \sum T(L_2) + \sum T(L_3) + \dots + (-1)^m \sum T(L_m) + \dots = 0$$

where $\sum T(L_i)$ represents the sum of transfer processes for all possible loops of order i that can be formed.

F. Application of Topological Equation in GERT

The random variable X_{ij} is the duration of the branch that starts at node i and ends at node j (i,j). From the given definition, the (i,j) activity could be realized only if node i could be realized. P_{ij} is the conditional probability that the activity (i,j) will be executed if node i can be realized.

$$W_{ij}(t) = P_{ij}M_{ij}(t)$$

G. Mean and Variance Calculation

Steps needed to determine the mean and variance value is as follows:

1. Using topological equations to obtain the W function that is equivalent to the GERT network..
2. Converting the W function into two parameters, which are the probability (PE) and the moment generating function (ME(t)) to the same work network time that is equivalent to the following equation:

$$W_E(t) = P_E \cdot M_E(t)$$

3. Determining P_E begins by changing the value of $t = 0$ and because the time parameter does not affect probability, then $ME(t=0) = 1$, so the following results is obtained:

$$W_E(t=0) = P_E \cdot M_E(t=0)$$

$$W_E(t=0) = P_E \cdot 1 \text{ thus } P_E = W_E(t=0)$$

$$M_E(t) = \frac{W_E(t)}{P_E} \text{ or } M_E(t) = \frac{W_E(t)}{W_E(t=0)}$$

4. Determining mean is carried out by finding the first derivative of ME (t) with $t = 0$ as follows:

$$\mu_1 = \left. \frac{dM_E(t)}{dt} \right|_{t=0}$$

5. Determining variance is based on equation (2.8); therefore, the second derivative of ME (t) with $t = 0$ is required as follows:

$$\mu_2 = \left. \frac{d^2 M_E(t)}{dt^2} \right|_{t=0}$$

$$\sigma^2 = \mu_2 - \mu_1^2$$

III. RESULT

PT XXX manufactures front and rear spar components for the wings of Airbus A321 aircraft. Both components are primary structural elements of fixed-wing aircraft that function to withstand static load and torsional forces, preventing the aircraft's wings from twisting during flight (Tariq & Mazhar, 2021). PT XXX often experiences delays in product completion and delivery to customers from the predetermined schedule. One of the determining factors causing this delay is the occurrence of rework processes arising from the need for highly precise components. Front and rear spar components have similar sequence of

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production processes. Both components have complex profiling process, which increase the likelihood of rework to meet the precise product specification. The rework process carried out include correcting the curvature and position of holes on the component in accordance to the specified tolerance. The company needs to design and re-establish the new production process timelines while also taking rework into account.

A. Production Activity Time Data

The time calculation on every activity and rework activity is obtained from direct measurement. Determining the shape of distribution time of every activity is done by statistically testing the time data that is collected using the Kolmogorov-Smirnov test.

B. Jaringan Kerja GERT

The GERT work network is constructed based on logical nodes and the sequence of production activity. The GERT work network also includes rework and reject activity. Every branch is sybolized by a transmittance that represents an activity. The presence of rework or reject probability create parallel paths in the network and dummy branches to help the network logic match the actual condition. The network is still in the form of an open network, therefore, an equivalent transmittance needs to be added to form a closed network. The network diagrams for the front and rear spar components can be seen in Figure 1 and Figure 2.

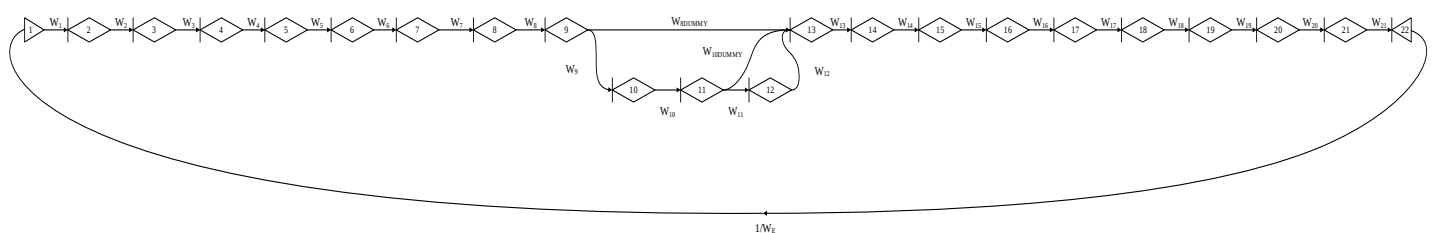


Figure 1. Closed GERT Network for the Front Spar

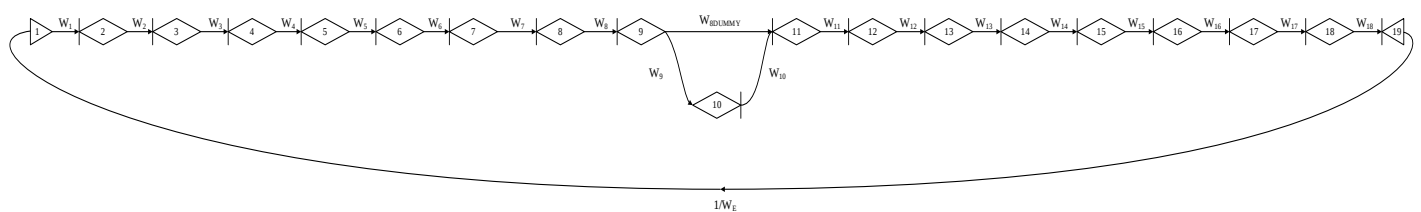


Figure 2. Closed GERT Network for the Rear Spar

C. Probabilitiy of Each Activity

The probability of each activity is obtained based on the amount of production carried out and the number of products that pass or fail to pass through an activity.

D. Fungsi Generator Momen and W Function

Based on the time distribution test data for each activity of both components, the distributions used in this study are the constant distribution and the normal distribution.

E. Looping

Looping data is obtained from the GERT work network that is already closed. The looping data for the front spar component work network can be seen in Table 1 and Table 2.

Table 1. Gert Work Network Looping Data For The Front Spar

Orde	Loop
L1	$L_1(1) = W_1W_2W_3W_4W_5W_6W_7W_8W_{8D}W_{13}W_{14}W_{15}W_{16}W_{17}W_{18}W_{19}W_{20}W_{21}1/W_E$
	$L_1(2) = W_1W_2W_3W_4W_5W_6W_7W_8W_9W_{10}W_{10D}W_{13}W_{14}W_{15}W_{16}W_{17}W_{18}W_{19}W_{20}W_{21}1/W_E$
	$L_1(3) = W_1W_2W_3W_4W_5W_6W_7W_8W_9W_{10}W_{11}W_{12}W_{13}W_{14}W_{15}W_{16}W_{17}W_{18}W_{19}W_{20}W_{21}1/W_E$

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Table 2. GERT Work Network Looping Data For The Rear Spar

Orde	Loop
L1	$L_1(1) = W_1W_2W_3W_4W_5W_6W_7W_8W_{8D}W_{11}W_{12}W_{13}W_{14}W_{15}W_{16}W_{17}W_{18}1/W_E$
	$L_1(2) = W_1W_2W_3W_4W_5W_6W_7W_8W_9W_{10}W_{11}W_{12}W_{13}W_{14}W_{15}W_{16}W_{17}W_{18}1/W_E$

F. Persamaan Topologi, Mean, dan Varians

The topological equations are determined based on the data loop in the GERT work network wshown in table 1 and table 2. These topological equation will be used in determining the W function that is equivalent to the entire GERT network. The equivalent W Function is used to calculate the mean and variance of the production time for each components. The calculation for the front spar is as follows:

- Topological equation and the equivalent W function

$$H = 1 - L_1 = 0$$

$$H = 1 - [L_1(1) + L_1(2) + L_1(3)] = 0$$

$$0 = 1 - \left(\frac{(W_1 \cdot W_2 \cdot W_3 \cdot W_4 \cdot W_5 \cdot W_6 \cdot W_7 \cdot W_8 \cdot W_{8D} \cdot W_{13} \cdot W_{14} \cdot W_{15} \cdot W_{16} \cdot W_{17} \cdot W_{18} \cdot W_{19} \cdot W_{20} \cdot W_{21}) + (W_1 \cdot W_2 \cdot W_3 \cdot W_4 \cdot W_5 \cdot W_6 \cdot W_7 \cdot W_8 \cdot W_9 \cdot W_{10} \cdot W_{13} \cdot W_{14} \cdot W_{15} \cdot W_{16} \cdot W_{17} \cdot W_{18} \cdot W_{19} \cdot W_{20} \cdot W_{21}) + (W_1 \cdot W_2 \cdot W_3 \cdot W_4 \cdot W_5 \cdot W_6 \cdot W_7 \cdot W_8 \cdot W_9 \cdot W_{10} \cdot W_{11} \cdot W_{12} \cdot W_{13} \cdot W_{14} \cdot W_{15} \cdot W_{16} \cdot W_{17} \cdot W_{18} \cdot W_{19} \cdot W_{20} \cdot W_{21})}{W_E} \right)$$

$$W_E(t) = \frac{(W_1 \cdot W_2 \cdot W_3 \cdot W_4 \cdot W_5 \cdot W_6 \cdot W_7 \cdot W_8 \cdot W_{8D} \cdot W_{13} \cdot W_{14} \cdot W_{15} \cdot W_{16} \cdot W_{17} \cdot W_{18} \cdot W_{19} \cdot W_{20} \cdot W_{21}) + (W_1 \cdot W_2 \cdot W_3 \cdot W_4 \cdot W_5 \cdot W_6 \cdot W_7 \cdot W_8 \cdot W_9 \cdot W_{10} \cdot W_{13} \cdot W_{14} \cdot W_{15} \cdot W_{16} \cdot W_{17} \cdot W_{18} \cdot W_{19} \cdot W_{20} \cdot W_{21}) + (W_1 \cdot W_2 \cdot W_3 \cdot W_4 \cdot W_5 \cdot W_6 \cdot W_7 \cdot W_8 \cdot W_9 \cdot W_{10} \cdot W_{11} \cdot W_{12} \cdot W_{13} \cdot W_{14} \cdot W_{15} \cdot W_{16} \cdot W_{17} \cdot W_{18} \cdot W_{19} \cdot W_{20} \cdot W_{21})}{W_E}$$

Thus the Equivalent W function is:

$$W_E(t) = 0,125e^{3.841,71t+1.049,73t^2} + 0,0625e^{4.241,57t+8.128,81t^2} + 0,8125e^{4.815,34t+19.623,22t^2}$$

- Determining ME(t)

$$P_E = W_E(t = 0) = 0,125 + 0,0625 + 0,8125 = 1$$

$$M_E(t) = \frac{W_E(t)}{P_E}$$

$$= \frac{0,125e^{3.841,71t+1.049,73t^2} + 0,0625e^{4.241,57t+8.128,81t^2} + 0,8125e^{4.815,34t+19.623,22t^2}}{1}$$

$$= 0,125e^{3.841,71t+1.049,73t^2} + 0,0625e^{4.241,57t+8.128,81t^2} + 0,8125e^{4.815,34t+19.623,22t^2}$$

- Calculating the mean value

$$\mu_1 = \left. \frac{dM_E(t)}{dt} \right|_{t=0}$$

$$= 4.657,77 \text{ minute/unit}$$

- Calculating the variance value

$$\mu_2 = \left. \frac{d^2 M_E(t)}{dt^2} \right|_{t=0} = 21.842.284$$

$$\sigma^2 = \mu_2 - \mu_1^2 = 147.463,63 \text{ menit}^2/\text{unit}^2$$

$$\sigma = \sqrt{\sigma^2} = 384,01 \text{ menit/unit}$$

The comparison results between the existing condition and the calculations using GERT can be seen in Table 3.

Table 3. Comparison of Time Between Existing and GERT

Components	Time	Standard Time	Existing Time	Calculation Using GERT
Front Spar	Production Time	3.841,69 Minutes/unit	4.251,35 - 5.064,15 Minutes /unit	4.657,77 Minutes /unit
	Completion Time 8 unit	25 Days	27 Days – 33 Days	30 Days
Rear Spar	Production Time	4.080,81 Minutes/unit	4.111,02 - 4.700,98 Minutes /unit	4.406,01 Minutes /unit
	Completion Time 8 unit	26 Days	27 – 30 Days	28 Days

IV. CONCLUSION

The completion time for both component calculated using GERT method is longer than the standard production time. This occurs because GERT method already takes into account non-steady condition in the actual situation, including the possibility of rework activities that increase the production time. The standard production time is the time used by the company to set the completion of production for both components, thus the completion time is smaller than the existing condition. The production time based on the existing condition is seen from actual production, including the rework processes, but it has not yet used calculation based on the GERT method. Calculation using GERT method produces a time span for completion times under existing condition. This indicates that the calculated production time are close to the actual conditions. The production time obtained from GERT method can be used by companies as a basis for scheduling and setting new completion times because they better align with existing condition.

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