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Comparative Performance of Penney-Price and Ref/Dif1 Models Validated by Laboratory Experiments for Wave Diffraction Through Breakwater Gaps

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ABSTRACT: Wave diffraction is a physical process in which wave energy is redistributed laterally along the wave crest when incident waves encounter coastal structures such as breakwaters. This process significantly influences both wave direction and wave height in the sheltered region behind the structure. To investigate wave diffraction through breakwater gaps with relatively small openings (less than five times the incident wavelength), a comparative study was conducted using numerical modelling and laboratory experiments. The numerical simulations were performed using the Ref/Dif1 model (Kirby and Dalrymple, 1994) and the classical Penney–Price diffraction model (Penney and Price, 1952), and the results were validated against laboratory measurements reported by Liu et al. (1999). The comparison indicates that both numerical models are able to reproduce the general diffraction patterns observed in the physical experiments. However, the Penney–Price model provides a closer agreement with the laboratory data in terms of the diffraction coefficient K_D, yielding a lower relative error of 16.16%, compared to 25.25% obtained using the Ref/Dif1 model. These results suggest that, for narrow breakwater gaps and constant depth, the Penney–Price model remains a reliable and computationally efficient approach for predicting wave diffraction.

KEYWORDS: Wave diffraction, Narrow breakwater gaps, Diffraction coefficient, Penney–Price model, Ref/Dif1, Laboratory validation.

I. INTRODUCTION

Wave diffraction is a key physical process in coastal engineering that occurs when incident waves interact with structures such as breakwaters, causing lateral redistribution of wave energy along the wave crest (Figure 1). This process modifies both wave height and propagation direction in sheltered regions, thereby influencing wave conditions inside harbors and port basins (Dean and Dalrymple, 1991). Accurate prediction of wave diffraction is therefore essential for harbor layout design and coastal protection. Diffraction effects are particularly pronounced in breakwater systems with gaps, which are commonly introduced to provide navigation access and improve water circulation. When waves pass through a gap, the gap edges act as sources of secondary waves, producing complex interference patterns behind the structure. For narrow gaps—typically defined as having widths smaller than five times the incident wavelength—diffraction dominates wave transformation, and small variations in geometry or wave conditions may lead to significant changes in transmitted wave heights (Penney and Price, 1952; Mei et al., 2005).

Laboratory experiments have long been used to investigate wave diffraction due to their ability to capture complex wave–structure interactions. However, physical modelling is often constrained by cost, scale effects, and limited parametric flexibility. Consequently, numerical models are widely employed as practical tools for diffraction analysis and preliminary design (Goda, 2010). The accuracy of such models depends strongly on their governing equations and numerical assumptions.

Although both models have been shown to reproduce general diffraction patterns, quantitative comparisons between them—particularly for narrow breakwater gaps—remain limited. Existing studies often focus on qualitative agreement or single-model validation without explicit error-based evaluation against laboratory measurements (Liu et al., 2000). Furthermore, the trade-off between model accuracy and computational efficiency is rarely addressed.

The present study addresses this gap by performing a comparative evaluation of the Penney–Price and Ref/Dif1 models for wave diffraction through narrow breakwater gaps (<5L). Both models are validated against laboratory experimental data reported by Liu et al. (2000), and their performance is assessed using the diffraction coefficient, K_D and relative error metrics. The results aim to clarify the applicability and limitations of each model and to provide practical guidance for model selection in breakwater design.

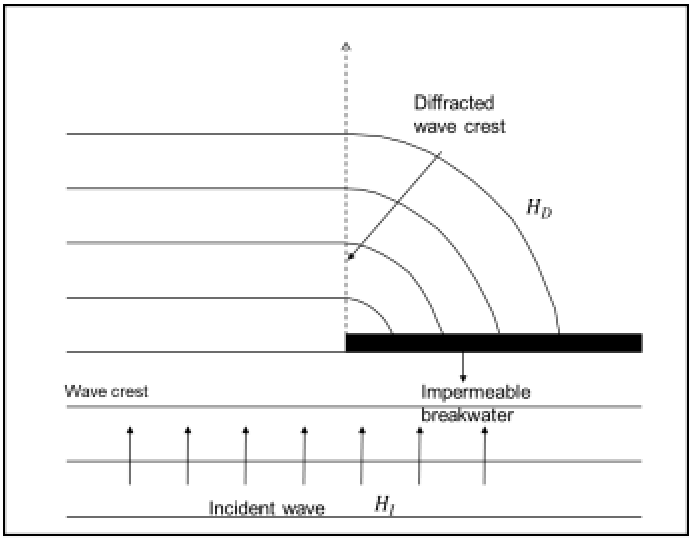

Figure 1. Definition Sketch of Wave Diffraction

II. NUMERICAL AND PHYSICAL DIFFRACTION MODEL

A. Penney-Price Diffraction Model

Wave propagation in the vicinity of coastal structures containing a gap represents a classical problem in wave hydrodynamics that cannot be adequately described using one-dimensional reflection and transmission concepts alone. When incident waves encounter a breakwater with a gap, part of the wave energy is not only transmitted directly through the opening but also spreads laterally into the sheltered region due to diffraction effects. To describe this phenomenon mathematically, Penney and Price (1952) developed a formulation based on linear wave theory, exploiting the direct analogy between sea-wave diffraction and optical wave diffraction.

Assuming an incompressible fluid and irrotational flow, the fluid motion is represented by a velocity potential $\phi(x, y, z, t)$ satisfying the Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

while the fluid pressure is given by the linearized relation

$$p = \rho(\phi_t - gz)$$

Applying the constant-pressure condition together with the kinematic condition at the free surface leads to the linear free-surface boundary condition

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial z} = 0 \qquad \text{at } z = 0$$

For a flat seabed with uniform water depth d, the impermeability condition at the bottom is expressed as

$$\frac{\partial \phi}{\partial z} = 0 \qquad \text{at } z = -d$$

Under these boundary conditions, the velocity potential is assumed in separable form as

$$\phi = Ae^{ikct}\cosh k(z + d)\, F(x, y)$$

where all geometric and diffraction effects associated with the coastal structure are contained in the complex two-dimensional function $F(x, y)$. Substitution of this form into the Laplace equation and the boundary conditions yields the governing equation for $F(x, y)$, which takes the form of the two-dimensional Helmholtz equation

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} - k^2 F = 0$$

The dispersion relationship between the wave number, *k* and the phase velocity, *c* is given by

$$c^2 = \frac{g}{k}\tanh\ kd$$

and the free-surface elevation is expressed as

$$\zeta = \frac{Aikc}{g\cosh\ kd}\ F(x,y)$$

Within this formulation, the modulus of *F(x,y)* directly represents the relative wave height, while the argument of $F(x,y)$ represents the wave phase.

B. *Ref/Dif1 Diffraction Model*

The Ref/Dif1 model is derived from a parabolic approximation of the linear wave propagation equations and is designed to simulate combined wave refraction and diffraction processes in coastal regions. The formulation is rooted in the mild-slope equation, which governs the spatial variation of wave motion over variable bathymetry and in the presence of coastal structures. In its general form, the mild-slope equation for harmonic waves can be expressed as

$$\nabla \cdot \left(CC_g \nabla \eta\right) + k^2 CC_g \eta = 0$$

where η is the complex free-surface elevation, k is the wave number, C is the phase velocity, and C_g is the group velocity. This equation accounts for both refraction due to depth variation and diffraction due to lateral wave spreading.

To improve computational efficiency, Ref/Dif1 adopts a parabolic approximation of the governing equation. This approximation was originally developed by Radder, who introduced a directional decomposition of the wave field by separating forward-propagating waves from backward-propagating waves. By assuming that wave energy predominantly propagates in one principal direction, the backward-propagating component can be neglected, resulting in a reduced parabolic form that is suitable for forward-marching numerical solutions.

Following the work of Mei and Tuck (1980), the parabolic equation was further simplified and applied to wave propagation around slender obstacles and gently varying coastal geometries. In this approach, the complex wave field is represented by a slowly varying complex amplitude $A(x, y)$, leading to the parabolic wave equation

$$\frac{\partial A}{\partial x} = \frac{i}{2k}\frac{\partial^2 A}{\partial y^2}$$

where x denotes the principal direction of wave propagation and y is the transverse coordinate. This equation retains the essential diffraction physics through the transverse second-derivative term while allowing efficient numerical marching in the x-direction. The complex amplitude A is directly related to the free-surface elevation, which may be expressed as

$$\eta = Ae^{i(kx-\omega t)}$$

where ω is the angular frequency. Spatial variations of A describe changes in wave height and phase caused by refraction, diffraction, and interaction with coastal structures.

The parabolic equation is solved numerically using a finite-difference scheme. In Ref/Dif1, the Crank–Nicholson method is employed due to its unconditional stability and second-order accuracy. This implicit scheme enables robust numerical simulation of wave refraction and diffraction over large spatial domains with reasonable computational effort. As a result, Ref/Dif1 has been widely applied in practical coastal engineering problems involving harbor layouts, breakwaters, and artificial islands.

Despite its versatility, the accuracy of Ref/Dif1 depends on the validity of the parabolic approximation. The model performs best when wave propagation remains predominantly forward-directed and when strong backscattering effects are limited. For narrow breakwater gaps, where lateral diffraction may dominate the wave transformation process, the performance of Ref/Dif1 requires careful validation against laboratory experiments, which is addressed in the present study.

C. *Diffraction Physical Model*

The experimental results used in this study are based on the laboratory experiments reported by Nadaoka, as presented in the paper Numerical Modelling of Boussinesq Equation by Finite Element Method by Y. S. Li, S. X. Liu, and G. Z. Lai (1999) as shown in Figure 2. In the experiments, a breakwater with a thickness of 0.35 m was installed parallel to the wave maker and positioned 7 m downstream from the wave generation system. The ends of the breakwater were shaped with semi-circular tips to reduce reflection and edge effects.

The measurement domain covered an area of 26 m × 27 m, within which a breakwater gap of width B=3.92 m was provided. Regular waves with a wave height of 0.05 m and a wave period of 0.92 s were generated during the experiments. These controlled conditions allowed detailed observation of wave propagation and diffraction patterns in the vicinity of the breakwater gap and provided reliable reference data for numerical model validation.

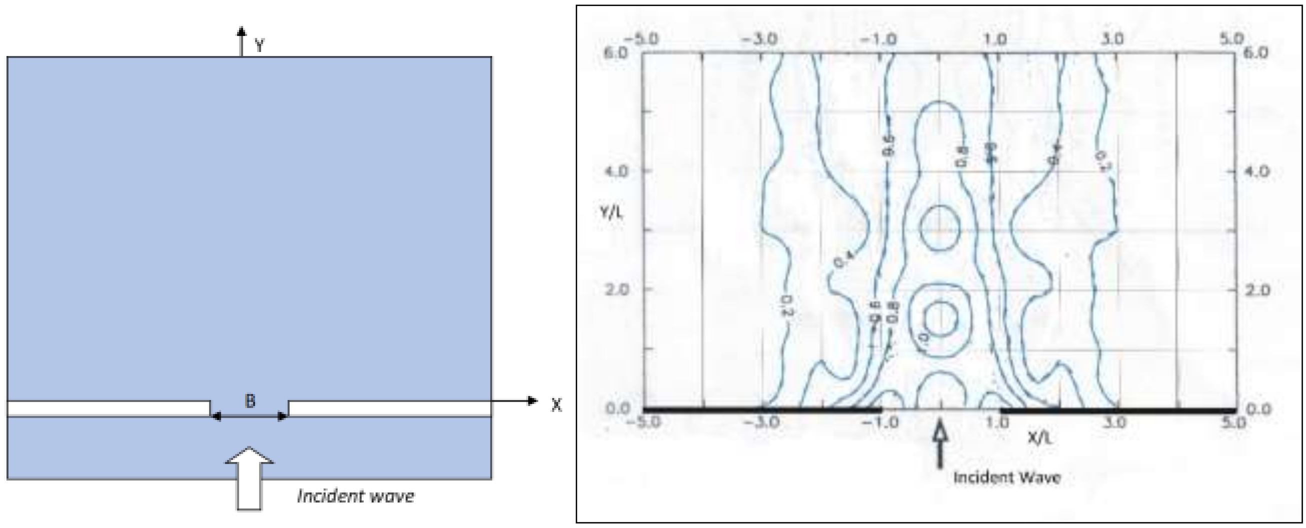

Figure 2. Description and Result of Physical Model (Y.S. Li et. Al, 1999)

III. RESEARCH METHODS

All This study employs a comparative numerical modeling approach to evaluate wave diffraction through narrow breakwater gaps. Figure 3 describe flowchart of research methods which two numerical models—the classical Penney–Price diffraction model and the Ref/Dif1 numerical model—are applied and validated against laboratory experimental results reported in the literature. The overall research procedure is structured to ensure consistency between numerical simulations and reference experimental conditions, allowing a direct and objective comparison of model performance.

The input data for the numerical simulations consist of the incident wave height H_i, wave period T, water depth d, and breakwater gap width b, selected to satisfy the narrow-gap condition (*B*<5*L*). These parameters are applied consistently in both numerical models.

Wave diffraction is first computed using the Penney–Price diffraction model, followed by the Ref/Dif1 numerical model under identical wave and geometric conditions. For each model, the diffraction coefficient $K_D$is calculated as the ratio of the diffracted wave height $H_d$to the incident wave height H_i.

The numerical results are then compared with published laboratory experimental data, and model performance is evaluated using relative error analysis to assess the accuracy of each approach.

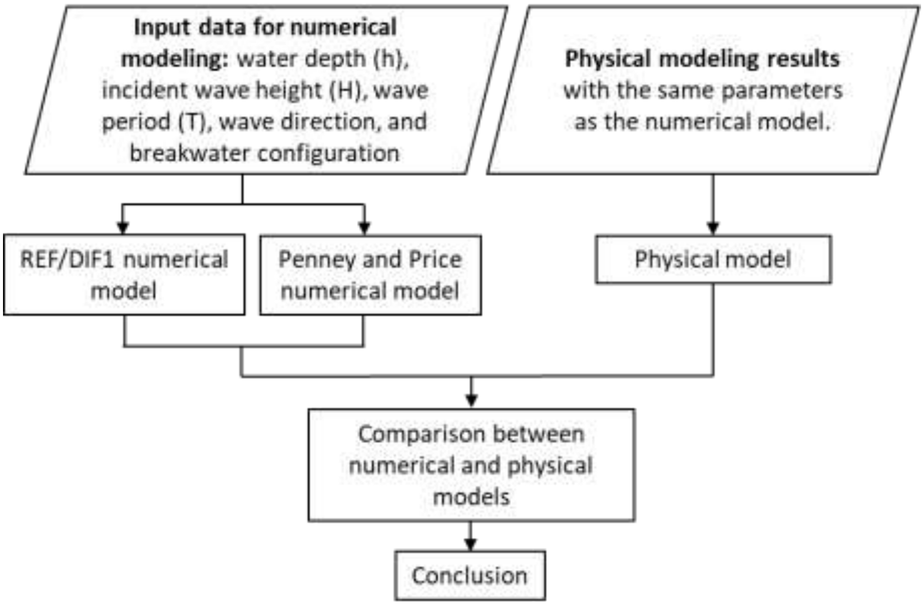

Figure 3. Research Methods

IV. RESULTS

From the Results of Penney–Price and Ref/dif1 numerical simulations for a breakwater with a gap width $B = 2L$, the following results are obtained:

- The diffraction coefficient contours produced by both models exhibit very similar spatial patterns.
- The average error in the diffraction coefficient K_D between the Ref/Dif1 and Penney–Price models is **21.58%**.

From the validation of the Ref/Dif1 model against the physical model experiments of Y.S. Li et al. for a breakwater with a gap width $B = 2L$ (Figure 4.), the following conclusions are drawn:

- The average error in the diffraction coefficient is **16.16%**.
- The largest average error occurs in the gap region (column D), reaching **25.25%**.

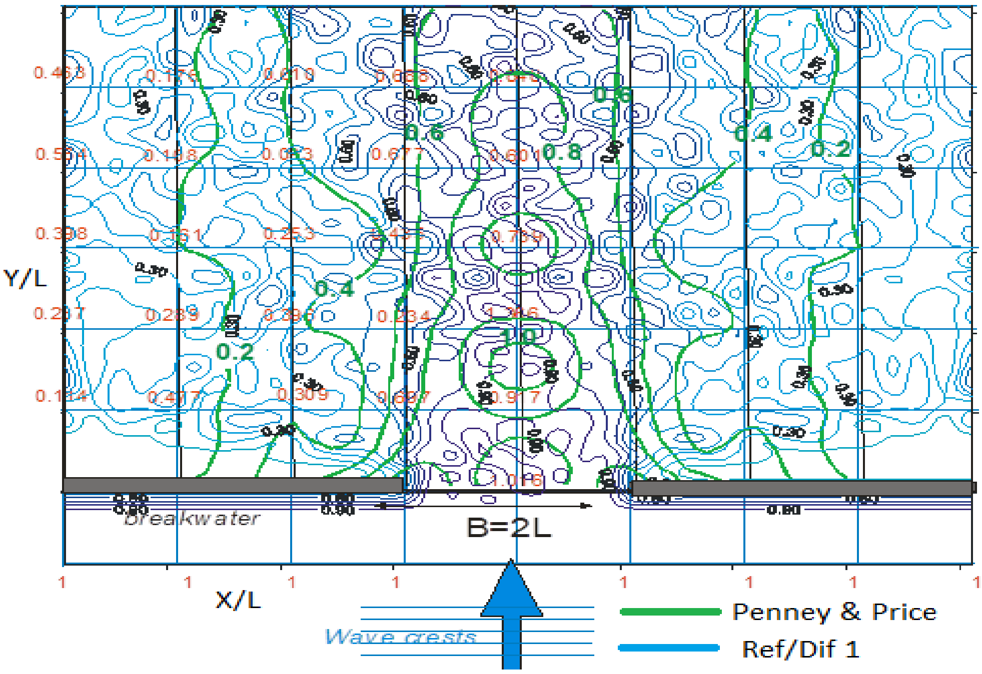

Figure 4. Comparison of K_D Values from the Ref/Dif 1 Model and the Physical Experiments of Y.S. Li et al.

From the validation of the Penney-Price model against the physical model experiments of Liu et al. for a breakwater with a gap width $B = 2L$ (Figure 5), the following conclusions are obtained:

- The average error in the diffraction coefficient is **7.8%**.
- The largest average error occurs in the gap region (column D), with a value of **12.28%**.

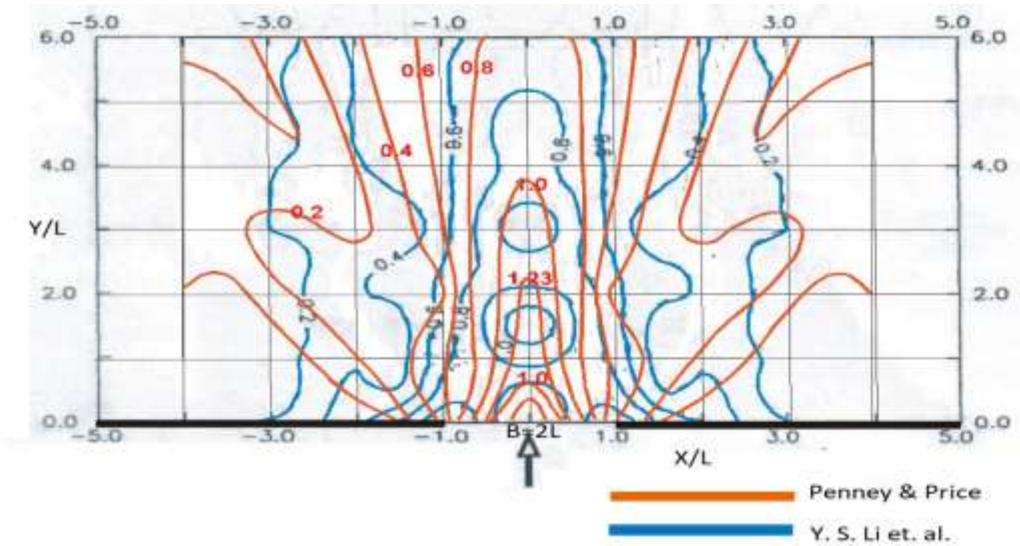

Figure 5. Comparison of K_D Values from the Ref/Dif1 Model and the Physical Experiments of Y.S. Li et al.

Overall, the Penney–Price (SPM-based) model exhibits smaller errors than the Ref/Dif1 model, indicating that the Penney–Price model provides better performance for wave diffraction analysis under the considered gap configuration.

IV. CONCLUSIONS

Both the Penney–Price method, which was later adapted into tabulated form in the Shore Protection Manual (SPM), and the Ref/Dif1 model are capable of accurately simulating wave diffraction through breakwater gaps. However, based on the error analysis and the diffraction contour patterns obtained under constant-depth conditions, the Penney–Price method shows closer agreement with laboratory experimental results than the Ref/Dif1 model.

V. ACKNOWLEDGMENT

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