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Brief Perspective on the Arrow of Time as Addressed in Terms of Simplicity Vs. Complexity Paradigm

Adam Gadomski

Department of Physics, Group of Modeling of Physicochemical Processes, Faculty of Chemical Technology and Engineering, Bydgoszcz University of Science and Technology, Bydgoszcz, Poland

ABSTRACT: The concept of the arrow of time is discussed in a perspective-type manner. The discussion is based on simple/complex physics-oriented examples, given in terms of irreversibility, symmetry breaking (or asymmetry), and certain machine-learning-type qualitative predispositions. Semi-quantitative solutions are sketched in a memory addressing manner, and with an extension to the original thought construct of Maxwell-demon-like or Szilard-type short-time-effective "intrusions" of 'minimally intelligent' quantum-classical beings/robots. Roughly speaking, when utilizing a fractional-time (memory-cost-involving) operator applied to the entropy production, nonequilibrium conditions would remedy certain problems based on small (and reversible) perturbation-in-time of typical fluctuational or (de)coherent behaviors, slightly affecting the course of the second law of thermodynamics as subjected to the minimally developed complexity of the phenomenon under consideration, also complemented by measurement-engaging intrinsic irreversibility. A multiscale thermodynamic course of the so-envisioned time's arrow is unfolded, touching upon its impact on a virtual (bio)system's behavior under consideration. The basics to be offered by the present work is thought to be a (short) memory-faculty-addressing mathematical replacement of the local operator (d/dt) by its nonlocal fractional counterpart (d^p/dt^p) for the entropy-production nonequilibrium conditions, using a fractional differential Caputo's operator in the so-called Lacroix algebraic representation, with a characteristic memory exponent, or quasi-allometry index, a $p<1$. The arguments allow to ascertain that fractional time derivatives, as the ones expressing a monotonous-in-time behavior, could be one way to uncover intrinsic irreversibility for the time's arrow as assessed by an open quantum-classical thermodynamic system undergoing the respective entropy course, conforming to the second law of thermodynamics.

KEYWORDS: arrow of time, complexity, entropy production, irreversibility, memory effect, second law of thermodynamics

INTRODUCTION

Physicochemical and biophysical systems, obeying fundamental laws of thermodynamics, either expressed classically or quantum-mechanically, may demonstrate a puzzling irreversible behavior, which is also involved in the concept of the arrow of time, as it was first addressed by Eddington in 1927 (Gujrati, 2024), and insightfully disclosed by Karl Popper in (Popper, 1956), who in 1956 published in Nature a short note on the arrow of time.

Despite a certain critique to be postponed here (Gujrati, 2024), in this short note (Popper, 1956), his first own example refers to an old and classical (thermo)mechanical system that demonstrates a reversible reference behavior as it would be observed for an ideal gas in a cylinder closed by a sufficiently tight (and ultralight) piston; for a subtle counter-example, with a time involvement, see (Gadomski et al. 2005). For systems of such a thermochemical nature, for which Newton's world can involve an arrow of time without having referred to contain irreversible processes, an increase of entropy is expected to occur. But whether it is expressed in terms of the Clausius depiction or that of the Boltzmann type, it does equally experience a proportionality to the logarithm of the ratio of final (V) and initial (v) cylinder volume values, namely $\ln[V/v]$, as long as the isothermal ideal-gas expansion for the closed Carnot-type system is concerned. Bear in mind that for the gas expansion, $V>v$ applies. Thus, the positivity of $\ln[V/v]>0$ occurs. This logarithmic measure is also accepted as the one mimicking an increase in measurement or observation time, since both volume magnitudes implicitly depend on time as they approach thermodynamic equilibrium. Of course, the bigger the temperature is, the greater the entropy $S\sim\ln[V/v]$ in the closed thermodynamic system appears to be.

However, after lowering the temperature (and, upon applying it still to the same system), the corresponding gas expansion does not develop, as the process must embark on its conversed volume-contraction mode, inevitably leading to a decrease of entropy in the form of $\ln[v/V]<0$, i.e., as attaining a negative value. But when recalling the reversibility, a simple mathematics inevitably yields ln[V/v] = - ln[v/V]. As argued properly by Popper in (Popper, 1956), this simple equality, when being referred to for the moment as the time (t) value, or in other words, as the arrow-of-time „signature", is mathematically very reminiscent of the Popper's retrospection relation t=-t. Popper even figuratively depicted it as putting into a film projector the last picture first (Popper, 1956). It is then worth noting that the gas in the cylinder is a statistical-mechanical system, irrespective of whether the N=1 (time-averaged) or the N>>1 (ensemble-averaged) circumstance is taken into account. The retrospection relation, if even slightly perturbed as such, can be expressed as a manifestation of the second law of thermodynamics (Koczan and Zivieri, 2024).

Then, Popper proposed in his short note an interesting non-film-projector-based example of a stone dropped on a quiescent water surface. He argued that the system tended to behave "non-classically", as for applying the retrospection relation t=-t that one had to make use of an auxiliary (supporting) concept of a vast number of coherent waves' generators. They have to serve for a certain coordination of the retrospective process towards the origin. However, bear in mind that the author did not make use of the fact that this phenomenon ultimately develops at the air-water by-density-distinguishable interface. Moreover, due to virtual air resistance, he was likely thinking of a „typically falling stone" but not of the one that is flat enough (light or "micro-gravitational" enough?) and would have a considerable aspect ratio, or for a pumice-stone-type, porous object, as if it would possess an appreciable surface-to-volume "colloid-type" ratio. A type of counter-example is proposed to be: What would happen to the thermodynamic (ir)reversibility when we are working with one water-containing phase only? That means, if we are going to put a cylinder-type stick into quiescent water, and then, when taking it gently out of the water, analyzing, however, the irreversible motion of water droplets forth and back toward the surface. If one continued this operation, while adjusting to the retrospection relation t=-t, exclusively, and when at the same time neglecting the remaining drops coming out (as if it would occur under shower conditions) from the in-motion but wet stick, and in addition, causing this way a type of hydrodynamic imbalance, or a certain water leakage (Gadomski et al., 2005), what will unavoidably happen to the otherwise reversible phenomenon under observation?

It turns out that Popper's note, being itself a concise thought-expressing harbinger, contains certain tacit assumptions, such as the ones, very typical of classical statistical thermodynamics, concerning a prominent account of the system's surroundings, or even at least some quite decisive physicochemical confinements. Namely, as it has ultimately been demonstrated by the last Popper's example of electromagnetism, and of Maxwell's theory, by quoting him herein explicitly (Popper, 1956): „one had to choose to work with retarded or advanced potentials", as one implicitly detected that the author did not wish to enter the whole within his work. The whole would imply the involvement of an external medium, diversity (or plethora) of physical scales, and/or engagement of the (virtual) confinement (Arango-Restrepo et al., 2025). When invoking the parlance of the complex-systems argumentation, he had foreseen that the whole is clearly not a simple „algebraic sum" of the parts, and on the contrary, as coined by Anderson, more is different (Anderson, 1972).

In what follows, a careful reader will be privileged to notice that the present article is going to address, to a pre-liminary but useful extent, the significance of fractional derivatives of Caputo type d^p/dt^p, and the by-fractional-derivative addressed entropy production for understanding the behavior of open (bio)physicochemical systems. The basic reason for making the fractional derivative operative for inherently chemical-in-nature systems is that a natural multiscale generalization effectively included in its definition accurately represents communication toward nonequilibrium channels, spanning over a wide range of space and/or time domains. The time domain appears to be of special interest here, at least the one characteristic of short duration (or the chemical system's reaction). This is because the measurements and/or observations of the thermodynamic-in-nature phenomena under discussion take the time required to capture the measurement's (or environmental) expression accurately.

To illustrate the overall article's scope and highlight the key concepts of this work, the reader is encouraged to refer to Figure 1.

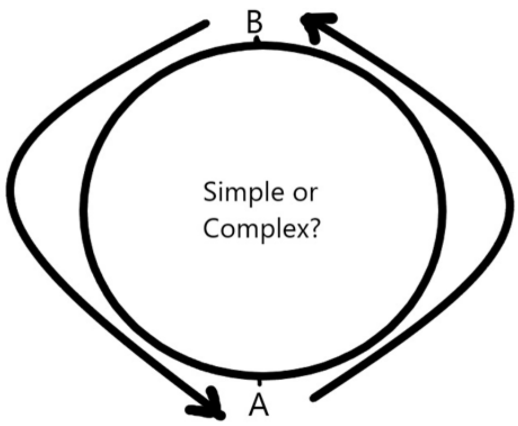

Figure 1: A Sketch of a Simple-Complex Diagram.

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Figure 1. The simple picture makes us aware that if the two ways from A to B and back are (bio)physico-chemically equivalent, then the system undergoes a reversible behavior of Popper's type, t=-t. If this is not fulfilled, it is conjectured that a temporal non-equivalence expressed by the system's fractional-time-derivative behavior may arise, especially because of an influence of the measurement and/or environment, or both. The latter factors render the system complex, and the expression of the passage between simplicity and complexity is recast by means of the suitable fractional derivative d^p/dt^p, wherein $0<p<1$ measures an extent between the former and the latter, corresponding essentially to a multitude of communication toward-nonequilibrium channels between the system and its surroundings, be it a measurement device or a part of the environment, or both.

Simple vs. Complex Scenarios as Rationalized by Methodological Examples

It ought to be clearly underlined that in this study, a semi-quantitative, maybe not directly explicit, mathematically supportive fractional calculus analysis is offered by the below presented argumentation line. In the author's opinion, it is assumed that the following rationale might be primarily understood as a thought-provoking study as aimed at prolonging the discussion. (In fact, this is also a short perspective article or, to some reasonable extent, an 'essay-like' perspective study, provoking to a topical discussion.)

As a suitable example, let us include in our further discussion the one that is illustrative of the cognition or decision-making sciences, thus, a (bio)physicochemical (Gadomski and Lent, 2017). Frankly, this complex example has been proposed as applicable to two distinctive nonlinear (albeit simple!) physico-chemical systems: The free fall and the (non)ideal-oscillator, the latter viewed more as the perturbation to the former. As we may accept it, the former can mimic a type of directional (gravitational or biased) action „toward destination spot" (figuratively, in a lose parlance of accepting a directional impact of a decision-making), while the latter could gladly mimic a model of "gravitational" hesitation (or, oscillating uncertainty) in decision-making or that of our (quite sensible) mood, or even of first mental-illness signature(s). In a crude macroscopic description, it is what the competing climbing fellows can do when vigorously climbing on a corrugated and obstacle-containing wall. Another far more complex instance would emerge when driving too fast a car through heavy-traffic conditions. All the obstacles and/or hindrances take a certain time to overcome. Both of the main physico-chemical phenomena invoked may at least touch a little upon disclosing certain neurological or psychological, memory-effect, including processes (Gadomski and Lent, 2017). However, to be honest, it is neither real nor seriously applicable, with the overall physical framework staying behind it. When inspecting this very circumstance a bit closer, one has to resort to escaping beyond the physical (free-fall and ideal-oscillator) laws, namely one ought to rely, at least in an additive manner (Gadomski et al., 2017), upon a random individual (speed-climber) „hesitation-time" finite reaction, caused by hazardous or „entropic", mostly external co-events or landscapes, as originating likely from a respective observation or suitable experiment (whether of laboratory or computer-simulation, or even of artificial intelligence viz machine-learning origins), to rationalize the reality encountered in the argued cognition behavior (Gadomski and Lent, 2017; Gadomski et al., 2017). Thus, the question of interest arises: Could one accurately reflect such a psycho-physicochemical, viz mental circumstance already introduced in terms of retrospection relation t=-t, i.e., reversibly, and, if yes, how useful can it be, in particular for a neuro-biomedical therapeutic and best-practiced, and carefully selected time-domain-addressing treatment (Gadomski et al., 2017)? In particular, whether a sensitively applied fractional calculus based on the d^p/dt^p operator would help model the psycho-biomedical or related circumstances of interest?

It is legitimate to address such a question in a model way, provided that the free fall can be simply defined by $dv/dt=-g$, with g being the acceleration due to gravity, and the ideal oscillator, supposed to be sensitively related with the free fall, can be classically expressed by $dv/dt+w^2 x=0$ and $v=dx/dt$, with x and v designating the displacement and the velocity, respectively, whereas w - the angular frequency of the oscillator. Notice, however, that for complexity modeling, d/dt can be replaced by its fractional counterpart d^p/dt^p, with $0<p<1$. Thus, as foreseen, the ideally detectable case ought to be that with p=1, holding for the ordinary derivative, whereas all nonideality, viz complexity, has to be attributed in principle to d^p/dt^p. The fractional parameter p is supposed to mimic in a simplified parametric manner the invoked communication channeling effects between the corresponding subsystems, and with the (sub)systems and the measurement device, and/or environment. Of course, the system can be thought of as either classical or quantum. In the latter, if p=1, the system is decoherent (close to classical); otherwise, the system would unveil a coherent behavior, and it is entangled with the external surroundings, thus, it is going to express an interference of communication of wave packets or channels (Zurek, 2022). Also, in a recent review on the concept of time reversal and the direction of time, a certain less orthodox view on this subject has been argued (López and Lombardi, 2024). This type of view would abandon to a reasonable extent the dynamics which are purely based on the standard Hamilton-Jacobi physical description. Instead, it is legitimate to offer certain non-orthodox (outside Hamilton-Jacobi realm) reliable propositions, such as those based on the fractional calculus, viewing the concept of direction of time as non-fundamental (non-congruent with the dynamics as developed in any time domain), but emergent (p-parameter dependent), rather. Similarly, in another study (Wolpert and Kipper, 2024), the so-called epistemic arrow of time has been introduced, the memory effects have been defined in terms of an adequate framework of the stochastic processes (Gadomski et al., 2017), and with a viable connection to the second law of thermodynamics; see also another study on the second law(s) of thermodynamics, revealing a multitude of quantum communication channels for an open thermodynamic system (Brandão et al., 2015).

Bear in mind that a thought-experiment illustrating scheme (Figure 1) on the paradigmatic (as in chemical reactions of (ir)reversible types) observation on when and why a path from a state A to a state B is, or is not (presumably, as in fractional order chemical reactions, in turn), equivalent to its B to A reversed counterpart can be sketched. An oversimplified, ideally circular pathway, going from state A to B, or conversely, is supposed to be realized under either simple or complex circumstances, although the semi-circular paths should then be substituted, depending on the fractional p parameter, and its actual range, by their arbitrarily (or freely) drawn (realistic) topological equivalents.

Interestingly, in a recent article (Seif et al., 2021), the authors discussed a back-and-forth flow of physical events in terms of asymmetry and the second law of thermodynamics, tightly associated with it, interpreting time's arrow as connected with the system's fluctuations, thereby preventing the differentiation of events' direction with certainty. The authors of (Seif et al., 2021) have qualitatively - mostly graphically - presented their relevant ideas as follows. First, they have outlined that the microscopically seen time's arrow concept does not show up for a macroscopic thermodynamic system because the thermal fluctuations do not influence its irreversible or „odd" entropy-production condition(s). Second, the irreversibility is far more easily manifested if the thermodynamic system becomes macroscopic. This seems to be obvious because the fluctuations are efficiently unfolded in a microscopic world, as it would be exemplified by the ratchet-and-pawl concept, as ingeniously invented by Feynman, and carefully discussed by others (Łuczka, 1999). As explicitly argued by (Seif et al., 2021), they foresaw, as it clearly seems to us, a relevant approach to disclosing time-asymmetry-involving peculiarities in complex-in-nature and Hamiltonian-driven nonlinear phenomena, e.g., (convolutional) neural networks, by resorting to the currently promoted techniques and methods of machine learning. However, they did not see any special need to identify the nature of the extended state space where the process is supposed to manifest (Gujrati, 2024).

Let us work further with this type of chemo-thermodynamic (albeit time involving; t) argumentation line by considering a controllable small (N=1)-thermodynamic system which can be a paradigmatic (Gujrati, 2024) one-particle (N=1) cylinder container divided by a very tight/ultralight (Gadomski et al., 2005) classically (but for a while not quantum-mechanically) impermeable membrane/piston, climbing up, thus, counter-gravitationally, and going down (conversely) by a negligible or measurable distance measure, i.e. by some height h(t), within a (small enough but finite) measurement time zone, t.

Preliminary Semi-Quantitative Results and Discussion

Suppose that with a probability P=1/2 (like in tossing a coin), the wandering classical/quantum particle can be found by an almost (albeit some time-consuming) non-intrusive measurement either above the membrane or beneath. Further suppose that it is detected by measurement or observation above the membrane (in a quantum Schrodinger-cat-type parlance: with an equal chance of either above or below the membrane in action), and the overall system is very slightly (in a very minimalistic manner) cooled (Szilard, 1929). In such a case, the ultralight membrane rises slightly. On the contrary, if the reverse thermodynamic action takes place, the piston/membrane goes slightly down. Overall, a converse scenario unfolds if the engaged testing and wandering quasi-particle can be detected in the lower part of the container, and the cooling/heating thermodynamic course, as in a (N=1)-particle Carnot engine principle applies (Koczan and Zivieri, 2024) The entire thermodynamic behavior of such a tiny engine is supposed to be reversible in terms of the discussed retrospection relation t=-t as far as the involved measurement conditions (Szilard, 1929; Aguilera et al., 2023) are making the system unperturbed, and presumably, standard-diffusion (symmetric) or not (thus, asymmetric) conditions tend to manifest (Metzler and Klafter, 2004).

To be a bit more specific, let us now invoke the system studied by Szilard so many years ago (Szilard, 1929). The system of a similar thermodynamic character as the one briefly unveiled above stands for a „deus ex machina" essentially (N=1)-device, uncovering its decrease in entropy, as if the system was permanently cooled by the intruding being or the invoked measurement/observation device. (At this stage of developing the rationale, let us remark that at the microscopic or quantum level of description, the Heisenberg uncertainty relations may specify the respective measurement conditions.)

To proceed further, let us introduce the entropy production rate (designated by e), namely e=dS/dt as applicable to an almost equilibrated state for the N=1 ideal (classical/quantum) gas system for which the equation of still non-equilibrated („ground") chemo-thermodynamic state reads as having reached the (local temporary) equilibrium, namely pV=kT, wherein, as in the gas-state equation, p, V, T have their usual meaning (k – Boltzmann constant), but they may depend (Szilard, 1929) on the measurement time t. As a consequence, the simplest presentation of the entropy production, e, but affected by even a "tiny measurement", may read e=k d/dt (ln[V(t)]), with d/dt representing the first-order (ordinary) differentiation (mathematically speaking, local) operator.

Because we are still left with the piston-cylinder, also one-particle conditions, therefore the (working subcylinder) volume V(t) is to be calculated exactly, and is given by V(t)=Ah(t), where the circular base with its surface area (of the cylinder) remains unchanged, A=const., and the „active" height h(t) is jiggling slightly either above or below the thermodynamic-equilibrium level of the piston's position, depending on how it reacts to the (slightly alternating) heating or cooling conditions, respectively; cf. see drawings of (Szilard, 1929).

To the best of the author's knowledge, it can be accepted (Aguilera et al., 2023) that such a system described above clearly bears a memory faculty (Szilard, 1929) associated with the measurement time (t). If this is taken acceptably, the temporal characteristic of e introduced above, via the integer-number differential operator d/dt, which „classically" is the first-order operator, yielding a classical result (Koczan an Zivieri, 2024) of positivity of e ~ k > 0; in addition, see the simple argumentation given below. As it follows from the second law of thermodynamics, it provides a temporal asymmetry, rendering the thermodynamic process irreversible (Metzler and Klafter, 2004), as if it would manifest in asymmetric ferromagnetic „memorizing" conditions (Aguilera et al., 2023). Otherwise, if the reversible conditions apply, the often-mentioned Popper-type relation t=-t prevails to hold.

To attempt to substantiate this complex type of argumentation, let us introduce quite on purpose the memory-faculty (or, t-measurement/observation) effect invoked above by putting it into e:=k d/dt (ln[V(t)]) with V(t)=Ah(t) and the obvious geometrical constraint of A=const. (Bear in mind, however, that if e=0 the equilibrium thermodynamic condition, or maximum-entropy state, is recovered (Koczan and Zivieri, 2024); thus, the microscopically expected reversibility condition t=-t prevails.) To the author's knowledge, the only essential, viz, productive and simple, namely finite-memory-involving modification to be employed here appears to be a memory-effect-emphasizing substitution of d/dt by d^p/dt^p - a fractional differentiation operator, primarily of Caputo type (Metzler and Klafter, 2004), with p being not equal to one, thereby exploiting a passage from time-locality to time-globality (or, nonlocality, at least), even if the latter time (measurement) zone is short, albeit finite. In fact, it is proposed to confine the Caputo fractional derivative to its Lacroix representation of power-law type expressed by the mathematical form of $d^p/dt^p\, F(t)$ with $F(t)=const.t^q$, where $d^p/dt^p\, F(t)=\Gamma(q+1)/\Gamma(q-p+1)\, t^{q-p}$, wherein Γ – the Gamma function, and for simplicity a "ballistic case" of $q=1$ is chosen, see discussion in (Gadomski and Kruszewska, 2012), rendering the ordinary-derivative and above introduced function in an equivalent form, dF/dt=const. (However, we will instructively, or simply for symmetry reasons, choose p=1/4;1/2;/3/4; the values are well-immersed in a fairly symmetrical way within the (0,1)-interval.)

It is then required to justify the usage of the non-local Caputo-Lacroix operator d^p/dt^p. First, although it is also linear, it bears by definition more information about the initial conditions of the process under study. When introducing any measurement toy into the system, it can capture even very tiny differences between the emission and absorption (presumably, quantum-mechanical, creation-annihilation) stages (Kastner, 2017). It would also work for small and even simple measurable systems that favor the arrow of time. This is because subtle types of intrinsic asymmetry-in-time expressing dynamics, putatively of (quantum) chaotic character, are imposed on the system, subjected to any peculiar measurement conditions in an adequate t-domain (Kastner, 2024; West, 2024).

For the „central" or symmetric case of the memory domain, p=1/2, i.e., when applying $d^{p=1/2}/dt^{p=1/2}$, one gets (Metzler and Klafter, 2004) that e ~ $t^{1/2}$ holds. (Notice that this way the positivity of entropy production, expressing the nonequilibrium state(s), is still assured because of t>0.) Since the (N=1)-system is still not properly equilibrated. But, it is very close to being such if it is exposed to a relatively short-time zone; it would supposedly behave as if it were in a (Einstein-Smoluchowski or Fokker-Planck-type (Gadomski et al., 2005) random-walk regime or within a purely diffusive stage, with the corresponding diffusion coefficient being constant at a given reference temperature T_r. Thus, it should be fairly accepted because the entropy production e is positive and proceeds within certain observation times' conditions (t-s) in a way very similar to the RMSD~ $t^{1/2}$ standard behavior (Metzler and Klafter, 2004) (RMSD – root-mean-squared displacement, proportional to a standard deviation of the Brownian viz Gaussian particle's statistics). As a consequence, the central-memory (p=1/2) entropy production e goes in a way reminiscent of a purely diffusive behavior, making a special „over-height-hesitation" to the impermeable but movable, classical membrane, jiggling up and down, but because of the measurement effect involved, being still unable to equilibrate this simple classically viewed thermodynamic system. Overall, the consequence of thermodynamic non-equilibration (Szilard, 1929; Metzler and Klafter, 2004) for the central, viz., symmetric index p=1/2 involving case may be attributed to a virtual, even, short, but finite-time measurement procedure employed, likely prone to certain chaotic initial conditions, as associated with the measurement per se (West, 2024). The index p in d^p/dt^p is supposed to presumably refer to a qualitative classification of the chaoticity type involved in a measurement, or can be associated with the emerging coherence (in general, quantum) states of the system (Zurek, 2022), thus, to be crudely ascribed to the temporary correlations emerging within the chemical system. It is left for a suitable judgment by an experimenter whether the system under examination can be envisaged as purely random-walk-like (p=1/2) or with some evident inclinations to sub- (such as that of p=1/4) or super-diffusivity (at p=3/4), see the discussion below.

For the two remaining, non-central circumstances of p=1/4 and p=3/4, but with the application of the fractional (integro-differential) Caputo operator (Metzler and Klafter, 2004; West, 2024), thus, when introducing d^p/dt^p to the entropy production term (e), one arrived at RMSD ~ $t^{3/4}$ and RMSD ~ $t^{1/4}$, respectively. (Technically speaking, the in-the-cylinder piston-height variations or "hesitations" should be as tiny as such that h(t) =1+x(t), with x=x(t) being a very small (pseudo-random) value, much less than one. Moreover, ln[1+x(t)] is supposed to be, because of its smallness, reasonably well approximated by x, and x=x(t) is assumed to go effectively as linearly proportional to t, but in a microscopic-ballistic or micro-gravitational conditions addressing short-time (t) manner, namely t_o << t; t_o is an initial instant. If such an argumentation line, involving an intruding being or an observation-biological being (Szilard, 1929), is taken for further consideration, the non-central and non-standard-diffusive temporal cases tend to violate (Metzler and Klafter, 2004) the reversibility (temporal) relation t=-t; thus, Popper's movie cannot be played back. If the argumentation is detailed somehow, the sub-case RMSD ~ $t^{3/4}$ unfolds (West, 2024), rendering the measurement-affecting (N=1)-process superdiffusive (with a very small heating agitation) (Metzler and Klafter, 2004), whereas for the sub-case RMSD ~ $t^{1/4}$ a subdiffusive (with a tiny cooling associated) classical scenario develops (West, 2024; Akimoto et al., 2018). Notice that the studied Szilard- or Maxwell-demon type (also, Boltzmann-type) thermochemical system, i.e., the (N=1)-system, with the volume V=V(t), accessible for the wandering particle, is supposed to be simple if the measurement or observation conditions within the very short t-domain applied are not solely involved, as they do not affect the thermomechanical equilibration conditions. Otherwise, it is thought to be complex when the very short, albeit fractional-integrated t-domain of performing the measurement(s) or observations matters, as it would be in the classical-quantum realm, quite purposely unveiled in (Gadomski, 2024). A seemingly far-distant but fairly robust analogy can be that of certain thermodynamic-kinetic phenomena of (bio)physical or cognition-addressed propensity (Gadomski and Lent, 2017; Gadomski et al. 2017) which would unfold with or without an intervention of our minds, or of our (mental) influence and/or consciousness's manifestation. It can be inferred that an analogy between a classical linear oscillator and a standard (at least classical) random walk is accepted to exist, and can be recalled here to illuminate the analogy between them both, but in a well-known context presented in (West, 2024). And, their very action resulted in the measurement networking procedure, as the one presented in (Seif et al., 2021) via a certain machine-learning protocol, as associated with (thermodynamic) nonequilibrium and time's arrow appearances.

As insightfully addressed by (West, 2024), the concept of replacing $d^{p=1}/dt^{p=1}$ by d^p/dt^p (with a fractional index p less than one; 0<p<1) is not entirely new. It likely first emerged from analytical considerations of the three-body problem performed by Poincaré, which culminated in the development of the chaos theory, as also manifested in physicochemically treated clustering phenomena (Gadomski et al., 2005). In the interesting article (West, 2024), it is first shown that complexity cannot be compatible with Newton's theory of dynamics, and that it should be complemented, for example, somehow by a suitable, rather qualitative, p-measure of chaoticity, or in a more quantum-classical border's parlance, of a (de)coherence (Zurek, 2022). This p-involving chaoticity/(de)coherence, subject to an "intruding" measurement (Szilard, 1929) can plausibly develop normally (p=1/2) or anomalously (e.g., p=1/4 – slowly; p=3/4 – vigorously).

In (West, 2024), one can also find, besides interesting historical chaos-addressing tours, a sketch of a useful fractional kinetic theory. It involves first Zaslavsky's arguments, leading to fractional Fokker-Planck equations (Gadomski et al., 2005; Metzler and Klafter, 2004) and fractal, or self-similar trajectories, representative of the chaotic dynamics. In particular, a detailed calculus for performing it can be found in (Metzler and Klafter, 2004). Some steps further in (West, 2024), one might reconcile the so-called allometry relations, characteristic of their allometry index (typically, less than one by its value; recall comparatively the fractional parameter p), disclosing that the functionality of the system (see our cognition addressing example below) is related by a power law with the system size, insofar the size is here attributed to as a signature of the t-domain (Gadomski, 2024). It is to say that the perturbation to the so-envisaged chemical system develops upon the corresponding, even the most tiny, let us say, delicate measurement time (t). Interestingly, also in (West, 2024), it was shown that the entropy production, denoted by e_W , would qualitatively be in accord with our results for the Caputo-calculated entropy production, e (see the beginning paragraphs of this section). Thus, $e \sim e_W$ would ultimately occur. The e_W, however, links together the subjective operational (Newton's) time, and its more objective, (bio)physicochemical time counterpart. In case of e, one would rely on a single "decoherent" (Zurek, 2022) measurement by means of the perturbation time, t.

A simplistic, quite general view of the entropy production (e) can also be presented below. Namely, if it is assumed that the departure from equilibrium is not far-distant (Onsager mode) (Gadomski et al., 2005; Gadomski, 2024) e=JX, wherein by definition J – the flux, and X – the "thermodynamic" force. The force X is always represented by (chemical-potential) gradient-type expression(s). The flux, in turn, represents the pace/speed, s, of the entropy production, typically s being proportional to some dv/dt, where v is a measure of the process's speed. When having the entropic (open) system prone to fluctuations (Łuczka, 1999) or promoting somehow (de)coherence (Zurek, 2022; Brandão et al., 2015), in other words, sensitive to the (measurable) environment, one shall freely embark on replacing d/dt in the above speed measure with some adjustable (and applicable to v) d^p/dt^p, wherein $0<p<1$.

Another subject, tacitly immersed in the current considerations, is what the complex system actually ought to be (Estrada, 2023), and how it is defined, as well as how it depends on the initial (chaoticity- or (de)coherence-expressing) conditions. Notice here that for this study, it remains to suffice that our complex system can be defined as a simple N=1-Carnot-type semiclassical engine, but with a t-lasting measurement. As we can take it for granted, the basic definition rests upon the inseparability of the parts (physical behavior vs. measurement), and the parts from the whole (Anderson, 1972), what would physically suggest a decisive involvement of the interactions (or, correlations viz intrinsic irreversibility conditions (Kastner, 2017; Chamberlain, 2024)) inside the system, as they can be quite complex, for example in superconducting ceramic (crystalline-amorphous) materials in which an assumption of the sensitivity of the phase transformations to characteristic time scales has also been explored by using the Caputo fractional derivative (Gadomski and Kruszewska, 2012). Intriguingly, and still within the present viewpoint expressed here semi-quantiatively, a minimal rotating Boolean thermodynamic system with (0,1)- or true-false (physical-computational) erasure acts can also support our general viewpoint (Bormashenko, 2022), this time in terms of dissipative conditions, involving smooth transitions within a reduced-dimensional system (Binder, 2023; Gadomski and Karpiński, 2025).

Concluding Remarks

The main outcome achieved by the present study permits a reasonable semi-quantitative inference within the presented fractional-derivative framework, leading to a discern between simplicity and complexity pools of the memory (t) involving (bio)physicochemical phenomenon (Akimoto et al., 2018), involving a model 'diffusive rejuvenation' process. However, to sum up conclusively, a certain type of questions involved and the open address revealed by the present study can be still formulated as follows: How could one effectively make use of the retrospection Popper-type relation t=-t as a starting platform (sic!) to employ it for comprehending some more complex systems' involving issues (see, refs. (Seif et al., 2021; Szilard, 1929), even such as those of cognitive (neural nets expressing) and psycho-physicochemical nature within a biological-time domain (West, 2024), t?

To conclude more specifically, and when oversimplifying (or exaggerating?) a little bit in an exemplary way: For the free fall, the basic („reducible") time variable always reads: $t=(v_f-v_i)/g$, wherein the (other) v-s denote the final and initial velocities, respectively, and g, as stated before, is a gravitational constant of the order of 10, measured in force-to-mass, namely, N/kg units). It is easy to see that by rearranging the ordering of v-s, resulting in having the „retrospective" time, in such a way that $t=(v_i-v_f)/g$, one gets t=-t, thus, having Popper's movie as played backwards (Binder, 2023). (It is always expected in macroscopic, crude physical experiments; cf. ref. (Gujrati, 2024; Popper, 1956) and recall partly the speed-climbing effect, viewed as a (bio)physicochemical phenomenon (Gadomski and Lent, 2017).)

By the way, in a microscopic world, a velocity reversal, as addressed many years ago by the Loschmidt paradox (apparently leading to a violation of the second law of thermodynamics), if the discussed memory effect is not seriously taken into account, cannot cause its violation, as claimed, for example, by (Binder, 2023).

However, it is not enough to describe a complex decision-making or cognition process, also that one invented by Szillard, since one had, at least, to complement the t variable by, say, another experimentally derivable „microscopic circumstance" addressing (another) time variable T, as to ultimately attempt approaching a real behavior by, for instance, a t+T, or t-T, or alike (Gadomski et al., 2017; Szilard, 1929; West, 2024). (However, realize that introducing T as either a random variable (Gadomski and Lent, 2017) or a quantum-physical (Gadomski and Karpiński, 2025) Heisenberg-uncertainty time-and-measurement variable, entailed with a complex-energetic and subdiffusion clustering effect (Chamberlain, 2024), cannot be excluded a priori.)

These are still intriguing, albeit open issues to be uncovered, or touched upon clearly and systematically, partly towards Anderson's sound assertion (Anderson, 1972) that more is different, or equivalently, that complexity matters (Estrada, 2024) – the latter being inherently entailed with the fluctuation (intrinsic irreversibility (Łuczka, 1999; Kastner, 2017)), Szilard-like (entropy-production) mode mentioned, and certain genuinely chaotic or (de)coherent effects (Seif et al., 2021; Łuczka, 1999; West, 2024). No doubt, it is very challenging to try embarking on quantum-mechanical effects upon having the system interacting with its environment, and under which circumstances the opposing, reversibility resembling arrows of time may be introduced just for acknowledging the present discussion (Guff et al., 2020).

Linking, similarly to the self-organization concept, entropy-production context (outside thermodynamic equilibrium or maximum-entropy conditions with fluctuations (Seif et al., 2021) or quantum-classical (de)coherence states (Zurek, 2022, Brandão et al., 2015)) appears to be a real challenge. Note in the meanwhile that employing the Caputo derivative (Gadomski and Kruszewska, 2012) in the context of maximum-entropy conditions yields properly the mathematical zero value or minimum energy signature viz, equilibrium point with (non)equilibrium fluctuations' mode, and the time reciprocity vs. complexity effect as primarily associated with the fractional parameter p differing from one (and, including the quantum-environment „clustering" conditions

(Gadomski et al., 2005; Chamberlain, 2024)) may uncover a novel, and fruitful pathway, in studying multiscale-interaction and dissipative (bio)systems' expressions with a broken-symmetry prevalence (Arango-Restrepo et al., 2025; Gadomski and Karpiński, 2025). By no means, however, can the Popper-type reciprocity or reversibility relation be treated here as a sole first-rank argument, supporting the fractional differentiation. Rather, it ought to be accepted as a starting level toward anticipating an irreversible and symmetry-breaking behavior. However, the symmetry-breaking (Arango-Restrepo et al., 2025) has to be dealt with (because of the apparent simplicity) more favorably in the time- than in the space domain.

To recap, the time's arrow (Popper, 1956; Chamberlain, 2024; Guff et al., 2025) is envisaged by the current study, as performed by relatively simple conceptual means/tools, of paving a subtle way of nonequilibrium (or time involving asymmetry (López and Lombardi, 2024)) thermodynamic stepping out from equilibrium towards first physical signs of departing from it, whether classically, or perhaps, quantum mechanically (Zurek, 2022). A modest pre-survey of (bio)physicochemical examples (Gadomski et al., 2017), of course, without pretending to its exhaustive character, is associated with this study to disclose its primary, fractional-derivative d^p/dt^p involving goal (as an example, also see (Gadomski and Kruszewska, 2012)) toward further discussion on the main time's arrow subject matter, in a style of simple or complex, that is the question. The first arguments developed, and the results following them, permit us to ascertain that fractional time derivatives (albeit in the purposely selected Caputo-Lacroix form) could be proposed as one way to uncover intrinsic irreversible chemical channels (somehow enchanted in d^p/dt^p) for the arrow of time as assessed by an open quantum-classical (chemo)thermodynamic system undergoing the second law of thermodynamics.

Intriguingly, the ratchet-and-pawl inrreversible-thermodynamic system, as first invented by Feynman, bears the intrinsic irreversibility conditions too (Gujrati,, 2024; Łuczka, 1999), as can also be seen in terms of arguments presented in (Kastner, 2017; Chamberlain, 2024).

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