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Modeling Covid-19 Deaths in Nigeria Using Exponential-Gamma Distribution

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ABSTRACT: This work examines the performance of Exponential-Gamma distribution and the existing Exponetial and Gamma in relative to the distribution that will best fit the data on Covid-19 death in Nigeria from March 2020 to April 2021. The parameters of the distribution were estimated using the method of maximum likelihood estimates. The newly developed Exponential-Gamma distribution was compared with the exiting Exponential and Gamma distributions using the log-likelihood function, Akaike information criterion (AIC), and Bayesian information criterion (BIC) as the criteria for selecting the best fit model. The results show that the newly developed Exponential-Gamma distribution performed better than the existing Exponential and Gamma distributions in contrast in terms of model fit, this showed that the newly developed Exponential-Gamma distribution is more flexible and precise in analyzing the Covid-19 data other than the existing Exponential and Gamma distributions.

KEYWORDS: Exponential-Gamma Distribution, Maximum likelihood Estimate, AIC, BIC, log-likelihood function, Covid19

I. INTRODUCTION

Statistical distributions are applicable majorly in describing real-world events. With respect to the suitability of these statistical distributions, their theory is largely studied and applied to many situations in real life. One of the many difficulties statisticians encounter when modelling natural life events is to be able to find an appropriate and efficient statistical distribution that will fit and model such events conveniently and more precisely. These statistical distributions posed substantial usefulness in modelling and analysing natural life phenomena that encompass uncertainty and riskiness.

Researchers in various fields of study have made use of probability distributions in modelling and analysing health data while numerous studies and research are constantly in progress while various attempts are being made to analyze and predict future trends of various diseases in refining plans for prevention and treatment of such disease occurrence. Below are brief literature reviews on the application of the statistical distribution on health data and other facets of life by different scholars such as Hisham et al (2021), Ayeni et al (2019) Ayeni et al(2020), Ayeni et al(2020), Hui et al (2011), Najm and Dhelal (2014) and Janus and Erik (2006). Therefore this study aimed to examine the performance of the newly developed Exponential-Gamma distribution by Ogunwale et al(2019) and the traditionally existing exponential and gamma distribution on Covid-19 data in Nigeria using the model selection criteria like the Akaike information criterion (AIC), Bayesian information criterion (BIC), and the log-likelihood function (l)

II. METHODS

The major concern of researchers in the modeling and analysis of health data is the interpretation of past health data regarding future probabilities of occurrences. Numerous probability distributions have been applied to various health data and have proved beneficial for health studies and surveys.

Analysis of most health data is largely determined by its distribution pattern. It has long been a recurrent topic of interest in the field of health in establishing a probability distribution that provides a suitable fit to most health data. In this study, we, therefore, aim to examine and fit the newly developed exponential-gamma distribution to Covid-19 death in Nigeria from March 2020 to April 2021. The method of maximum likelihood will be used to estimate their parameters, while the log-likelihood, Bayesian information criterion (BIC) and Akaike information criterion (AIC) goodness of fit test will be employed to determine their goodness of fit, and Python software would be used for data analysis.

The new Exponential-Gamma distribution was developed by Ogunwale *et al* (2019) and its pdf is defined as

$$f(x;\alpha,\lambda)=\frac{\lambda^{\alpha+1}x^{\alpha-1}e^{-2\lambda x}}{\Gamma(\alpha)},x,\lambda,\alpha>0 \quad (1)$$

They defined its mean and variance as;

$$\mu=\frac{\alpha}{2^{\alpha+1}} \quad (2)$$

and $$V(x)=\frac{\alpha\left(\alpha 2^{\alpha}-\lambda\alpha+2^{\alpha}\right)}{\lambda\left(2^{2(\alpha+1)}\right)} \quad (3)$$

The cumulative distribution function is defined as

$$F(x)=\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)} \quad (4)$$

The survival function for the distribution defined by $S(x)=1-F(x)$ was obtained as;

$$S(x)=1-\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)} \quad (5)$$

While the corresponding hazard function defined by $h(x)=\frac{f(x)}{S(x)}$ was obtained as;

$$h(x)=\frac{\lambda^{\alpha+1}x^{\alpha-1}e^{-2\lambda x}2^{\alpha}}{2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)} \quad (6)$$

The cumulative hazard function for distribution is defined by

$H(x)=W(F(x))=-\log(1-F(x))\equiv\int_0^x h(x)dx$ and was obtained as;

$$H(x)=\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)} \quad (7)$$

A. *Maximum Likelihood Estimator (MLE)*

Let $X_1,X_2\ldots X_n$ be a random sample of size *n* drawn from a probability density function $f(x;\theta)$, the likelihood function is defined as;

$$f(x_1,x_2\ldots x_n;\theta)=\prod_{i=1}^{n}f(x_i;\theta) \quad (8)$$

Now, we estimate the parameters of both the exponential-gamma distributions using maximum likelihood estimation as follows, If $X_1,X_2,\ldots,X_n$ be a random sample of size n from Exponential-Gamma distribution. Then the likelihood function is given by;

$$L(\alpha,\lambda;x)=\left(\frac{\lambda^{\alpha+1}}{\Gamma(\alpha)}\right)^{n}\prod_{i=1}^{n}x_i^{\alpha-1}\exp\left(-2\lambda\sum_{i=1}^{n}x_i\right) \quad (9)$$

by taking the logarithm of (9), we find the log-likelihood function as;

$$\log(L)=\alpha n\log\lambda+n\log\lambda-n\log\Gamma(\alpha)+(\alpha-1)\sum_{i=1}^{n}\log x_i-2\lambda\sum_{i=1}^{n}x_i \quad (10)$$

Therefore, the MLE which maximizes (10) must satisfy the following normal equations;

$$\frac{\partial \log L}{\partial \alpha} = n\log\lambda - \frac{n\Gamma^{'}(\alpha)}{\Gamma(\alpha)} + \sum_{i=1}^{n}\log x_i \qquad (11)$$

$$\frac{\partial \log L}{\partial \lambda} = \frac{\alpha n}{\lambda} + \frac{n}{\lambda} - 2\sum_{i=1}^{n} x_i \qquad (12)$$

The solution of the non-linear system of equations is obtained by differentiating (10) with respect to (α,λ) gives the maximum likelihood estimates of the model parameters. The estimates of the parameters can be obtained by solving (11) and (12) numerically as it cannot be done analytically. The numerical solution can also be obtained directly by using python software using the data sets.

III. RESULTS AND DISCUSSION

This work used secondary data collected based on the total monthly amount of deaths of covid-19 patients in Nigeria from the period of March 2020 to April 2021. The data was collected from the website of the Nigeria Disease Control (NDC).

The newly developed Exponential-Gamma distribution was applied to the data to determine its flexibility in modeling such data. The data were analyzed using the Python software package. The flexibility and the measures of goodness fit were determined by using selection criteria such as the Akaike information criterion (AIC), Bayesian information criterion (BIC), and the log-likelihood function (ℓ). The probability distribution with the least Akaike information criterion (AIC), Bayesian information criterion (BIC), or the highest log-likelihood function (ℓ) value will be considered the most flexible and best fit model, and the results are obtained as follows;

TABLE 1: SUMMARY OF THE DATA

STATISTIC	ESTIMATE
n	428
mean	275.0667
skewness	1.484541
kurtosis	2.249087

Interpretation

The results from table 1 above indicated that the distribution of the data is skewed to the right with skewness of 1.484541. Also, it was observed that the kurtosis is 2.249087 which is lesser than 3. This implies that the distribution of the data has a shorter and lighter tail with a light peakedness when compared to that of the Normal distribution.

TABLE 2: THE MAXIMUM LIKELIHOOD ESTIMATES FOR THE DISTRIBUTIONS

Distribution	**Exponential-Gamma**		**Exponential**	**Gamma**	
Parameters	**Shape(α)**	**Scale(λ)**	**rate(λ)**	**Shape (α)**	**Scale(λ)**
Values	1.830997	1.0064	1.31417	1.02403	148.5737
Standard Error	0.021411	0.02312	0.1756164	0.2343093	40.4559698

TABLE 3: THE LOG-LIKELIHOOD VALUE FOR THE DISTRIBUTIONS

Goodness of Fit	**Exponential-Gamma**	**Exponential**	**Gamma**
Log-Likelihood	**-18.09301**	-25.03405	-83.89466

Interpretation

The log-likelihood value for the data is presented in Table 3. It was observed that the Exponential-Gamma distribution provides a better fit as compared to the existing Exponential and Gamma distributions since it has the highest value of log-likelihood (ℓ). Hence, the Exponential-Gamma distribution performed better than the existing Exponential and Gamma distribution.

TABLE 4: THE AIC AND BIC VALUE FOR THE DISTRIBUTIONS

Goodness of Fit	Exponential-Gamma	Exponential	Gamma
AIC	**52.3014**	172.0681	151.7893
BIC	**58.25578**	178.70716	153.067

Interpretation

The AIC value for the data is presented in Table 4. It was observed that the Exponential-Gamma distribution provides a better fit as compared to the existing Exponential and Gamma distributions since it has a smaller value of AIC while the BIC value for the data also presented in Table 4 showed that the Exponential-Gamma distribution provides a better fit as compared to the existing Exponential and Gamma distributions since it has a smaller value of BIC. Hence, the Exponential-Gamma distribution performed better than the exiting Exponential and Gamma distributions.

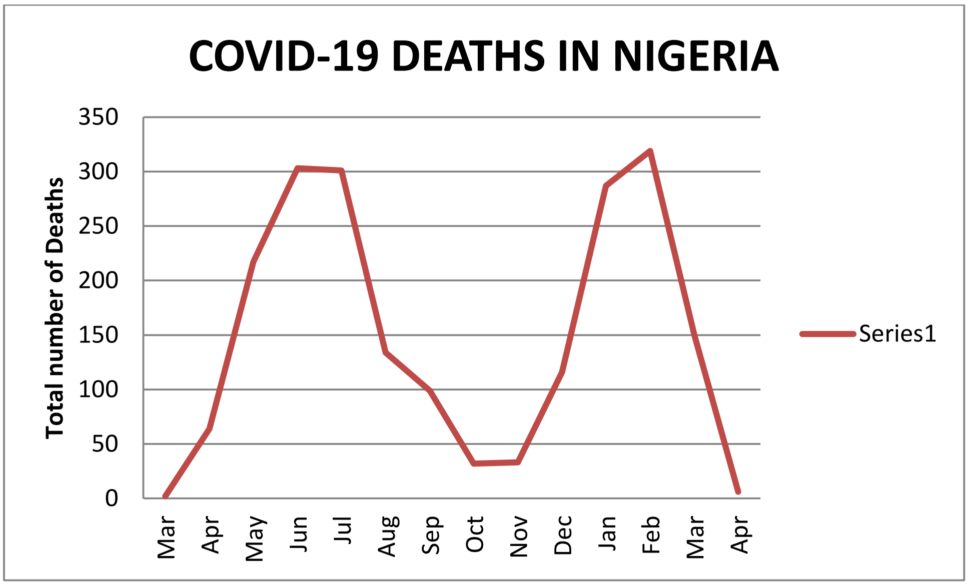

FIG 1: The Graph Showing the Total Number of Covid-19 Deaths

Figure 1 above shows the trend of Covid-19 from March 2020 to April 2021. It was observed from the graph that the virus rises to its peak between June and July 2020 but dropped to a lower rate from August to November 2020. The virus again from December 2020 to March 2021 experienced a higher rate while it flattens to the lowest in April 2021.

V. CONCLUSIONS

Having completed all the required analysis and tests, it becomes crucial to make the following inferences; The model with the lowest AIC and BIC value and also the model with the highest log-likelihood value respectively was selected; it was discovered that the newly developed Exponential-Gamma distribution is noticeably low and high in values in term of the AIC, BIC and log-likelihood values respectively. Thus, the quality of fit of the Exponential-Gamma distribution was confirmed without differentiating superiority. Equally, the appropriateness of the Exponential-Gamma distribution for life data analysis has been shown in advanced studies. Therefore, for flexibility and greater precision in analyzing Covid-19 death data, the use of Exponential-Gamma distribution is greatly suggested and also persistently useful in various fields where analysis of health data is essential.

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