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Variation of Linear Algebra Problems, Based on Algebraic Models, In the Training of Teachers of Mathematics at Univerisdad Autonoma De Santo Domingo UASD

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ABSTRACT: This report deals with one of the presented problems in the initial training of mathematics teachers and aims to characterize the process of variation problems of linear algebra at the Autonomous University of Santo Domingo. The methods used were the historical-logical and the analysis-synthesis. The main results are the precision and systematization of different theoretical approaches proposed by authors and the conclusive analysis of the process.

KEYWORDS: Technique, construction of problems, problem variation, linear algebra, future Math teacher.

I. INTRODUCTION

In the opinion of some authors, such as Gracia (2012) cited by (Pérez, 2018), students for mathematics teachers have not received training in the necessary skills, or in the techniques that allow them to model real situations, which limits the formulation and resolution of problems related to Linear Algebra. This problem is particularly evident in the Dominican Republic, which is why it has been one of the main reasons for carrying out this research.

Day by day technological changes and advances occur in the treatment of the different spheres of knowledge at a global level, which cause challenges in the different actors that affect the transmission of knowledge. These changes, in turn, affect the teaching-learning processes, so it is concluded that current education must adapt to the new demands imposed by the great scientific and technological advances in a globalized society that requires being prepared to interact in all sectors: economic, social and cultural; hence the concern of many countries for the training of teachers, due to the important role that they play in the solid construction of knowledge in future generations. (Pérez, 2018).

Hence, the objective of this study is: to design a technique to obtain a linear algebra problem, from a given problem of the same nature, on the basis of algebraic models**.**

II. METHODOLOGY

The methodology used combines different methods and techniques of didactic study, in it the method of analysis - synthesis is combined to systematize concepts related to linear algebra problems and their construction; the modeling method is used to design a technique of variation of linear algebra problems based on algebraic models; and participant observation is applied to a course that inquiries about the validity of the proposed technique.

III. PROCEDURE

To achieve the proposed objective, it is necessary to previously characterize the variation of linear algebra problems and clarify some conceptual assumptions. Among these are those of: mathematical problem, some classifications thereof, the structure of a problem, as well as the characteristics of linear algebra problems and their models.

Main Characteristics of linear algebra models and problems

In the first place, the meanings of the concept of a problem, in particular a mathematical problem, of different authors who have ventured into this subject will be considered. here are many challenges for the training of teachers in mathematics, the great didactic trends in mathematics training, prioritize including the vertiginous advance of technology in the entire learning process as a powerful tool to contribute to improvement of performances. For countries like the Dominican Republic and in general today face new opportunities and challenges in all areas of life, many of which stem from the rapid deployment of computers and devices like robots, smartphones, and networked machines. For example, most young adults and students who started university post 2015 have always considered phones to be mobile hand-held devices capable of sharing voice, texts, and images and accessing the internet.

The big question then arises for all the institutions in charge of training teachers WHAT TO TEACH, AND HOW TO TEACH, many restrictive understandings arise from the way mathematics is conceived. Many people see mathematics as no more than a useful toolbox. A clear trace of this approach can be found in the school curricula of many countries. This perspective on mathematics is far too narrow for today's world. It overlooks key features of mathematics that are growing in importance.

On the other hand, Cruz, M. (2002) makes a synthesis of what was expressed by other authors, and quotes them, to accept as a problem that situation that is characterized by the existence of a person (or group) who wants to solve it, of a state initial to final, and some kind of impediment to the passage from one state to another. This author starts from a general definition of a problem that he later uses in the conceptualization of their formulation for the teaching of Mathematics.

In the case of González, D. (2001) accepts the definition of Labarrere, A. (1994) of a mathematical problem with text and adds two elements, not explicit in it, which Campistrous, L. and Rizo, C. (1996) refer to.), that is: (1) the solution path must be unknown; (2) the person must want to solve the problem (motivation). Definition that could be accepted for the purposes of this research but that its name is redundant and excludes certain types of problems.

Malaspina, U. (2007), states that a mathematical problem is any situation that requires analyzing information, establishing logical relationships, and drawing conclusions. Concept that is very open to be accepted as a presupposition in this research, although it achieves some precision when it later describes its fundamental components.

In the most recent specialized literature, the following definition has been found: a mathematical problem is defined as a statement that contains an unknown situation of interest to a subject that requires mathematical means for its solution (Pérez, K. 2017). As this definition is not in contradiction with those analyzed in this section, it will be taken as a reference and will be used to make a synthesis of the structural parts of a mathematical problem.

A mathematical problem consists, at a first level of the statement and of the mathematical means. The statement, in turn, consists of an initial situation and an unknown final situation (quantitative or qualitative) to find, examine, or conclude. The initial contains the fact or phenomenon under investigation, the context where it occurs and the data (quantitative or relational). For its part, mathematical means are made up of the environment and the way of resolution. The environment is formed by the mathematical concepts, their relationships and practical meanings that intervene or may intervene to solve the problem. And the way is given by the procedures, methods and strategies used in its solution. These components, indistinctly, are distinguished by authors such as: Polya, G. (1976); Davidson, L and Reguera, R. (1987); Labarrere, A. (1988); Llivina, M. (1999) González, D. (2000); Malaspina, U. (2007); and Pérez, K. (2017), were later recreated in this research from the assumed problem definition.

In the case of linear algebra problems, the environment is that part of mathematics related to this branch. The present investigation preferably refers to problems whose context is extra mathematical.

If the result of the problem is taken as a basis from the relationships present in the data, then linear algebra problems can be classified - regardless of context - into consistent and nonredundant; inconsistent and consistent and redundant. The inconsistent ones present a contradiction; therefore, the result is the empty set, or it is false. Consistent and nonredundant have only one solution. Redundant consistencies can be redundant with respect to the relationships and / or variables present in the data. When the relationships are redundant it means that some of them are left over, and when the variables to be investigated are redundant it means that there are missing relationships. However, a valid result can be obtained in either type. Furthermore, it is said that a problem is well formulated algebraically if it is consistent and not redundant. Finally, it can be stated that two linear algebra problems are equivalent if they have the same number of variables in the final stated situation and have the same solution set.

An apparently simple example of the types of problems is presented in those that lead to the expression $ax = b$. If a is different from zero, then the problem is consistent and not redundant; if $a = 0$ and $b \neq 0$ is inconsistent; and if $a = 0$ and $b = 0$ it is consistent and redundant. It is significant that the theories of linear algebra allow, in most cases, to classify problems without reaching the solution, if certain characteristics of the corresponding model are known. This case constitutes a potentiality from the functional point of view for the treatment of problems in the PEA of Linear Algebra.

Regarding algebraic objects and their relationship with the context, it is corroborated by what has been stated by Pecharromán (2013), who considers that the reason for the existence of mathematical objects is to be or represent a functionality in a context, associated with the nature that originate them. It is concluded that mathematical objects, in particular those of linear algebra, acquire a specific identity as a result of their functionality in the context that gives them origin, which is subsequently developed in other spaces of use. The functionality of a mathematical object is given by its potential to be used in different contexts and each new use endows it with new meanings.

On the other hand, Casas and others (2012a) pose five types of problems in the context of linear algebra, a classification that was made with the purpose of creating a knowledge base in the computer sense. As in this research emphasis is placed on problems where the context is extra-mathematical, these problems are then built into mathematical models where the objects used to represent the facts or phenomena are typical of linear algebra and that in this research will be simply referred to as algebraic models. Each of them has its equivalent in each of the theories that make up this branch of Mathematics, a fact that favors different environments for each problem and different ways of interpreting them. The authors consider that these entities contain a potentiality, from the functional point of view, to interpret the changes that can be made in the problems of linear algebra.

In this section, the following were specified: the definition of the mathematical problem to be considered in this research, some classifications of the same and the parts that compose it. Also, the main characteristics of linear algebra problems and their models were described, highlighting their functionality for the former.

Variation of linear algebra problems

There are many mathematical relations that occur in life. For instance, a flat commission salaried salesperson earns a percentage of their sales, where the more they sell equates to the wage they earn. Because the field of research is relatively new and little explored, the epistemological characterization of the variation of linear algebra problems will be done through the contextualization of some types of construction of mathematical problems. In principle, the construction of mathematical problems is understood to be any process of obtaining them.

For Malaspina (2013), the variation of a mathematical problem is conceived as a process according to which a new problem is built, modifying one or more of the fundamental elements of the given problem. The fundamental elements are understood as the information (the quantitative or relational data that are given in the problem); the requirement (what is requested to be found, examined or concluded, which can be quantitative or qualitative); the context (it can be intra-mathematical or extra-mathematical); the mathematical environment (the global mathematical framework in which the concepts that intervene or may intervene to solve the problem are located).

In this investigation, the variation of linear algebra problems is considered as a process in which one problem (of linear algebra of course) is built from another by transforming one or more components of the given problem. This process, as can be seen, refers to the context of linear algebra and differs from that defined above by the referents relating to mathematical problems and their components, which were specified in the previous section.

In another section, Malaspina (2013a) refers to the variation processes; and elaboration of mathematical problems as creation processes, based on the definition of mathematical problem creation given by the authors Stoyanova and Ellerton (1996). In addition, it considers that elaboration is a process of creation according to which a new problem is constructed, either in a free form, from a situation (given or configured by the author), or by a specific request, thus establishing the difference. between both processes, only in terms of the starting point. But there are indications to think about another difference, and that is that the variation of problems is a productive process, although it could become creative, as will be explained later.

The variation; and the elaboration; of problems have the same objective, and that is for the student to enunciate mathematical problems with precision, in which their referents reflect that part of the historical-cultural heritage of humanity that the student wants to show; that is, with meaning for him. (Pérez, K. 2017). This is the interpretation of significant mathematical problems considered by the authors.

The fundamental difference between those processes; it lies in the mathematical tasks (exercises, problems or situations) that demand their completion. In the case of variation, they generally contain in themselves a higher level of help than that presented in the elaboration, because in the first, one starts from a problem that must be transformed into another and in the second, the point of departure is only a fact or situation. This aspect suggests that it is necessary to consider the levels of assimilation of the content required for the realization. In the opinion of the authors of this research, the task on variation of linear algebra problems can and should be considered as an open problem, like other processes of construction of mathematical problems (see Cruz, 2002). This assertion is made based on the potentiality that, from the functional point of view, the objects of this branch of Mathematics present; specifically when the algebraic model of the problem object of variation is formed, as explained in the previous paragraph. In this type of task, the requirement is to state a new problem (final situation) with certain characteristics based on a given problem. The problem to be obtained is not unique (each student can offer a different answer) hence the open nature of the task.

From the psychological perspective of Vîgotsky's (1987) historical-cultural approach, one of his main contributions consists of the notion of the “Zone of Proximate Development” (ZPD), which expresses the relationship between education and development. This concept is characterized by the need for an asymmetric novice-expert relationship, as the genesis (in the former) of higher psychological processes; and also by the appearance of a potentiality, which emerges from this relationship. Here the genetic law of development is manifested, which postulates that every psychic process appears twice: first on an interpersonal plane, then on the intrapersonal plane

En el presente trabajo se es consecuente con esta noción de ZDP, en la cual cada estudiante debe trabajar sobre las fronteras de su propio conocimiento. Consecuentemente, la tarea sobre la variación de problemas no debe ir orientada hacia el nivel actual de desarrollo del escolar, sino hacia la ZDP. La situación inicial de la tarea planteada debe estar concebida para el nivel actual, pero la situación final debe conducir a que el estudiante se apropie de un nuevo conocimiento. Aquí debe ponerse de manifiesto la relación asimétrica novato–experto donde el estudiante es guiado por sí mismo a partir de las orientaciones que le ofrece el profesor. Es por ello, que se deben considerar dos aspectos esenciales: uno subjetivo y otro objetivo. Esto permite comprender que la tarea que exige la variación de problemas será un problema si el paso del estado inicial al estado final implica que el estudiante experimente un desarrollo cognitivo, al trabajar sobre su ZDP. Con esta descripción se comprende que este tipo de tarea puede ser un problema para un estudiante y para otros puede no serlo, esta relatividad constituye una alerta para el profesor, quien debe estar pendiente cuándo una tarea sobre la variación de problemas deja de ser un problema para el estudiante para convertirse en un ejercicio o problema “rutinario”. Esta denominación ha sido utilizada por otros autores, entre ellos al reconocido Pólya (1957).

- You can take advantage of the functionality of linear algebra objects, specifically algebraic models, to carry out transformations in problems and to characterize the transformed problem.
- The linear algebra tasks corresponding to the variation of problems can and should be considered as problems.

Variation of linear algebra problems based on algebraic models

The present work is consistent with this notion of ZPD, in which each student must work on the frontiers of their own knowledge. Consequently, the task on the variation of problems should not be oriented towards the current level of development of the student, but towards the ZPD. The initial situation of the proposed task must be conceived for the current level, but the final situation must lead the student to appropriate new knowledge. Here the asymmetric novice-expert relationship should be revealed where the student is guided by himself from the guidance offered by the teacher. That is why two essential aspects must be considered: one subjective and the other objective. This allows us to understand that the task that requires the variation of problems will be a problem if the passage from the initial state to the final state implies that the student experiences a cognitive development, when working on her ZPD. With this description it is understood that this type of task can be a problem for a student and for others it may not be, this relativity constitutes an alert for the teacher, who must be aware when a task on the variation of problems ceases to be a problem for the student to become a "routine" exercise or problem. This name has been used by other authors, including the renowned Pólya (1957).

- You can take advantage of the functionality of linear algebra objects, specifically algebraic models, to carry out transformations in problems and to characterize the transformed problem.
- The linear algebra tasks corresponding to the variation of problems can and should be considered as problems.

Variation of linear algebra problems based on algebraic models taking into consideration the theoretical constructs previously analyzed, a technique for the variation of linear algebra problems can be inferred that consists of:

Given a linear algebra problem (initial problem) build the corresponding algebraic model (initial model). Then make modifications, as appropriate, to the initial model to arrive at a new model (resulting model). And finally state a problem (resulting problem) that corresponds to the final model. This process is schematically represented in the following Figure.

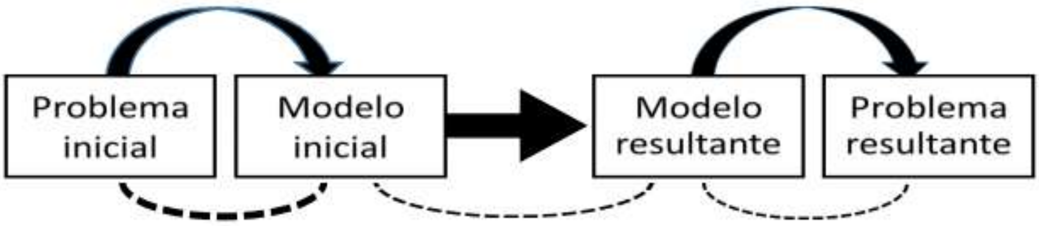

Figure 1. Variation technique of linear algebra problems based on algebraic models.

Experimental course at the XXXIII Latin American Meeting of Educational Mathematics. (RELME 33)

The workshop consisted of explaining to the students the theoretical basis of a well formulated problem and a redundant problem, based on the theoretical assumptions of the works on production of linear algebra problems by (Pérez, 2018). The idea was that students could vary a given problem in light of its components.

To carry out the workshop, two problems formulated by (gracia, 2012) were adapted and which resulted in the following way:

Problem 1. A recycling company uses discarded paper and fabric to make two different types of recycled paper. A batch of grade A recycled paper is made from 40 kg of fabric and 160 kg of paper, while a batch of B-type recycled paper is made from 80 kg of fabric and 120 kg of paper. The company has 280 kg of cloth and 960 of paper. How much recycled paper of each type can be made with 280 kg of fabric and 960 kg of paper?

Problem 2. A bottler uses three kinds of pure juice - pineapple, orange and lime - to make two mixed juices, pineapple-orange and pineapple-lime, which are sold in 1/2-liter cartons. Each mixture is obtained by mixing equal amounts of each of the juices that make it up. The amount of juice available is 80 l. of pineapple juice, 50 l. of lime juice and 30 l. of orange juice. How many cartons of each juice mix can be made?

Participants were informed that the first is well formulated algebraically and the second is redundant. And then the participants were asked to perform the following tasks:

For the first:

1. Formulate an equivalent problem and justify your answer.
2. Justify the fact that it is well formulated algebraically and for this use the following concepts.
   a) The SEL classification. b) The rank of a matrix. c) The determinant of a square matrix. d) Base of a real vector space. e) The kernel and the image of a linear map.
3. Formulate an extended problem to three types of recycled paper such that the new problem:
   a) Have a solution.
   b) There is no solution.

For the second:

4. Formulate a new problem where the amount of juice available varies so that the new problem:
   c) Have a solution.
   d) There is no solution.
5. Formulate a new problem where you vary the proportions of the mixture.
6. Formulate an equivalent problem where redundant information is removed

As results of the workshop, the students were able to vary the problem and came quite close to the objectives of the workshop, at first it was very difficult for them to vary the given problem, presenting difficulty on the basis of the content involved in the dynamics, but then they were clarifying the doubts that have arisen until the problem and its model are varied.

IV. CONCLUSION

The theoretical and practical analysis described in the body of this report showed the need to coherently articulate the assumptions of the approach to the construction of mathematical problems initiated by Malaspina (2013, 2015) with the teaching-learning approaches of Linear Algebra, in the search for a didactic model that gives a coherent explanation to the variation of linear algebra problems as a content application process.

The exposed technique considers the realization of transformations in the algebraic models, which is why it is a process that requires the integral systematization of the contents. This implies the need to consolidate the application of the content to diverse situations, as a way of promoting the construction of problems by the student.

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